Observational Astronomy - Spring 2014
Homework 4 - Orbits, Motions, and Kepler's Laws
Answer Sheet

1. Halley's comet has an orbital period of 76 years. It was last seen in 1986, and is due again in about 2061. Based on Kepler's third law of planetary motion, what is the semi-major axis of Halley's comet's orbit in astronomical units (AU)? The eccentricity of the orbit is known to be 0.967. What is the perihelion distance (closest approach to the sun) in AU? What is the aphelion distance (furthest distance from the sun) in AU? Since the intensity of the light from the sun varies as the inverse square of the distance from the sun, how much more intense is the sunlight falling on the comet at perihelion than at aphelion? Show your calculations.

   - Kepler's third law says that $a^3 \propto T^2$. If we measure a in AU and T in years, then the constant of proportionality is 1, and we can write that $a^3 = T^2$. Since we are given that T = 76 years for Halley's comet:
   $$a = \sqrt[3]{T^2} = \sqrt[3]{76^2} = \sqrt[3]{5776} = 17.9 \text{ AU}$$

   - The perihelion distance is given by:
   $$R_P = a(1 - e) = 17.9(1 - 0.967) = 0.59 \text{ AU}$$

   - Similarly, the aphelion distance is given by:
   $$R_A = a(1 + e) = 17.9(1 + 0.967) = 35.2 \text{ AU}$$

   - The ratio of sunlight intensity is given by the inverse square of the ratio of these two distances:
   $$\frac{I_{\text{Perihelion}}}{I_{\text{Aphelion}}} = (\frac{R_A}{R_P})^2 = (\frac{35.2}{0.59})^2 = 3560$$

   - This is why comets put on such a showy display when they get close to the sun, because the intense sunlight boils off liquids and gases that remain frozen when the comet is far from the sun. The resulting cloud of gas and dust is what makes the comet's tail.

2. Explain why you will never see Venus in the sky at midnight if you live in New York.

   - At midnight we are looking out away from the sun. But Venus' orbit is entirely inside the Earth's orbit, so it is hidden by the body of the Earth. Venus and Mercury are only ever visible near the sun in the sky, so just after sunset in the evening or just before sunrise in the morning.

3. Mercury's orbit has a semi-major axis of 0.387 AU, and an eccentricity of 0.206. What is the maximum distance of Mercury from the Sun? Based on this, what is the maximum angular separation in degrees of Mercury from the Sun, as seen from Earth? An accuracy of $\pm 1°$ is good enough for this.

   - The maximum distance of Mercury from the sun is given by:
   $$R_A = a(1 + e) = 0.387(1 + 0.206) = 0.467 \text{ AU}$$
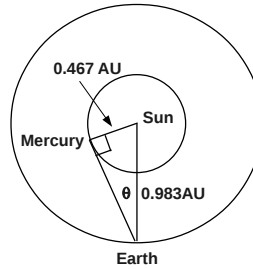
- Looking at the figure, the maximum angular separation of Mercury from the sun is when Mercury is at aphelion and the Earth is at perihelion, although the Earth's orbit is circular enough that just using 1.0 AU for the Earth's distance will easily get you within the requested $\pm 1°$:

$$\theta = \arcsin(\frac{0.467}{0.983}) = \arcsin(0.475) = 27.8°$$

- If we use the small angle approximation, we would get:

$$\theta = \frac{0.467}{0.983} \text{ radians} = 0.475\frac{180}{\pi} = 27.2°$$

- So this angle is getting large enough that the small angle approximation introduces a small error.

4. The planet Venus has a radius of about 6000 km. What is the angular size of Venus in arcseconds at inferior conjunction and at superior conjunction? You can assume that the orbits of Earth and Venus are both circular and that the orbit of Venus has a semi-major axis of 0.72 AU. Remember that an AU is about 1.5×10^8 km.

   - At inferior conjunction the distance to Venus will be 0.28 AU, and at superior conjunction it will be 1.72 AU. Using the small angle approximation, the angular sizes will be:

$$\theta_{\text{Inferior}} = \frac{2 \times 6000 \text{ km}}{0.28 \text{ AU} \times 1.5 \times 10^8 \frac{\text{km}}{\text{AU}}}\frac{180}{\pi}3600 = 59''$$

$$\theta_{\text{Superior}} = \frac{2 \times 6000 \text{ km}}{1.72 \text{ AU} \times 1.5 \times 10^8 \frac{\text{km}}{\text{AU}}}\frac{180}{\pi}3600 = 9.6''$$

5. The Earth has a semi-major axis of 1.0 AU (by definition) and an orbital eccentricity of 0.017. Mars has a semi-major axis of 1.524 AU and an orbital eccentricity of 0.093. The closest approach of Mars to the Earth is called *opposition*, where Mars lies on the meridian at midnight. However, the distance from the Earth to Mars at opposition can vary significantly because the orbits are not circular. How close is Mars to the Earth at its closest opposition? How about at its its furthest opposition?

   - The perihelion and aphelion distances of the two planets are given by

$$R_{\text{P−Earth}} = a(1 − e) = 1.0(1 − 0.017) = 0.98 \text{ AU}$$

$$R_{\text{A−Earth}} = a(1 + e) = 1.0(1 + 0.017) = 1.02 \text{ AU}$$

$$R_{\text{P−Mars}} = a(1 − e) = 1.524(1 − 0.093) = 1.38 \text{ AU}$$

$$R_{\text{A−Mars}} = a(1 + e) = 1.524(1 + 0.093) = 1.67 \text{ AU}$$

- The closest opposition is when the Earth is at aphelion and Mars is at perihelion, so the distance is 1.38 - 1.02 = 0.36 AU. The furthest opposition is when the Earth is at perihelion and Mars is at aphelion, so the distance is 1.67 - 0.98 = 0.69 AU. So it is almost twice as far away at the furthest opposition as at the closest.

6. You've probably heard that there are plans to send colonists on a one-way trip to Mars. Let's design an orbit to get them there. The lowest-energy way to travel from one planet to another is using what is called a "Hohmann transfer orbit". To get from the Earth to Mars, this is an elliptical orbit that just touches Earth's orbit at perihelion, and just touches Mars' orbit at aphelion. So it has a perihelion distance of 1.0 AU, and an aphelion distance of 1.524 AU. What is the eccentricity of such an orbit? What is its semi-major axis in AU? Based on Kepler's third law, what is the orbital period of this orbit? Based on this, how long will it take our colonists to travel from the Earth to Mars?

    - Since:
    $$R_P = a(1 - e)$$
    $$R_A = a(1 + e)$$

    - We can write:
    $$a = \frac{R_P + R_A}{2}$$

    - and:
    $$e = \frac{R_A - R_P}{R_A + R_P}$$

    - So:
    $$a = \frac{1.0 + 1.524}{2} = 1.26 \text{ AU}$$

    - and:
    $$e = \frac{1.524 - 1.0}{1.524 + 1.0} = 0.208$$

    - Based on Kepler's third law:
    $$T = \sqrt{a^3} = \sqrt{1.26^3} = 1.41 \text{ years}$$

    - This is the period for the whole orbit, but traveling from the Earth to Mars is only one-half the orbit, so it takes 0.7 years or about 8 and one-half months.