Observational Astronomy - Spring 2014
Homework 3 - Telescopes and the EM spectrum
Answer Sheet

1. Name two reasons why we put telescopes in space.

   - Possible answers are:
     - There is no air, so no atmospheric turbulence.
     - Air absorbs most wavelengths of the EM spectrum, but they are all visible from space.
     - There are no clouds to block the view.
     - You can observe all the time, not just at night.

2. Name two different components of the electromagnetic spectrum that you use every day.

   - Possible answers are:
     - Visible light.
     - Radio waves for your television, radio, or cell phone.
     - Microwaves in your microwave oven.
     - ...

3. Which has a longer wavelength, blue light or red light?

   - Red light.

4. Assume you are at a dark-sky site and can see with your naked eye stars down to magnitude +6. You pull out your trusty 7x50 binoculars, which have objective lenses 50mm in diameter. How many times more light do they gather than your naked eyes, which have pupils 7mm in diameter? What is the faintest magnitude you can see in the binoculars? Show your calculations.

   - Pupil of eye $\approx$ 7mm in diameter in dim light, binoculars = 50mm in diameter.

   $$\text{Area ratio} = \frac{\pi(50 \text{ mm})^2}{\pi(7 \text{ mm})^2} = 51.02$$

   $$m_{\text{Binoculars}} - m_{\text{Eye}} = 2.5 \ \log_{10}(\frac{I_{\text{Binoculars}}}{I_{\text{Eye}}}) = 2.5 \ \log_{10}(51.02) = 4.27$$

   So you can see down to about m = 6.0 + 4.3 = 10.3.

5. The Hubble Space Telescope has a mirror D = 2.4m in diameter, and has diffraction-limited optics with resolution given by $\theta = 1.22\lambda/D$. It orbits the Earth about 560 km high. Suppose we pointed it at the ground and looked in visible light with $\lambda = 500$nm. Could it resolve individual people, about 2m in size? Could it read street signs, with letters about 50cm high? Could it read your smart phone, with letters about 5mm high? Show your calculations.

- The Hubble space telescope has an angular resolution of about 0.05 arcseconds, as calculated by:

- λ is the wavelength of visible light $= 500$ nm $= 5.0 \times 10^{-7}m$

- D is the diameter of the mirror $= 2.4$ m

$$\theta = 1.22\frac{5.0 \times 10^{-7}\text{m}}{2.4\text{m}} = 2.5 \times 10^{-7} \text{ radians}$$

$$\theta = 2.5 \times 10^{-7} \text{ radians} \times \frac{360}{2\pi} \times 60 \times 60 = 0.05 \text{ arcseconds}$$

Remember that an object of size R at a distance d (I used small d this time) subtends and angle in radians of $\theta = $R/d, assuming the small angle approximation. So, since the angular resolution $\theta = 0.05$ arcseconds, and d $= 560$ km, the size of the object R that can be resolved is give by:

$$\text{R} = 2.5 \times 10^{-7} \times 560 \text{ km} = 2.5 \times 10^{-7} \times 560000 \text{ m} = 0.14 \text{ m} = 14 \text{ cm}$$

So you could see people and read street signs, but you couldn't read your smart phone.

6. There are many very exciting discoveries lately of exoplanets, which are planets orbiting stars other than the sun. We would like to build a telescope to see features on these planets. Suppose we would like to resolve continent-sized objects 1000km across in visible light with $\lambda = 500$nm on a planet orbiting a star 10 parsecs away. One parsec is 3×10^{16}m. How large would the diameter of the telescope need to be, assuming that the resolution is determined by $\theta = 1.22\lambda/$D? Show your calculations.

- To resolve an object 1000 km across at a distance of 10 parsecs requires an angular resolution of

$$\theta = \frac{1000 \text{ km} \times 1000\frac{\text{m}}{\text{km}}}{10 \text{ pc} \times 3 \times 10^{16}\frac{\text{m}}{\text{pc}}} = 3.3 \times 10^{-12}$$

In visible light, this requires a mirror such that:

$$\theta = 3.3 \times 10^{-12} = 1.22\frac{5.0 \times 10^{-7}\text{m}}{\text{D}}$$

Solving for D:

$$\text{D} = 1.22\frac{5.0 \times 10^{-7}\text{m}}{3.3 \times 10^{-12}} = 150000 \text{ m} = 150 \text{ km}$$

So it would take a mirror almost 100 miles across! We won't be building a mirror like this any time soon, although there is a technique calles interferometry that may allow us to come close to this someday.