Observational Astronomy - Spring 2014
Homework 1 - Coordinates and Angles
Answer Sheet

1. Why do we use angles to determine the position of objects in the sky?

   - We cannot easily tell the distance to celestial objects, we can only tell the direction. Different directions are quantified by giving two angles between two reference directions and the object we are interested in.

2. You are looking at the Statue of Liberty from a distance. If you extend your arm out your thumb is just the right size to cover the height of the statue. Are you in Flag Plaza, Governor Island Picnic Park, Battery Park? (Show your calculation, measure the length of your arm and your thumb beforehand, and google the size of the Statue of Liberty, if you do not know it. Eventually, you can help yourself with, for example, Google Maps).

   - Note that your numbers may be slightly different, since your thumb and arm are different from mine. My thumb is about 2 cm wide, and my arm is about 50 cm long. So, with my arm outstretched, my thumb subtends and angle of $\arctan(2/50) \approx .04$ radians. Note that the small angle approximation would give you the same answer, since $2/50 = .04$. Looking up the height of the Statue of Liberty, it is 305 feet or about 93 m or .093 km tall. So the distance from where I'm standing is D = .093 km/.04 = 2.3 km. Looking at a map, this puts me in Battery Park.

3. Why do multiple coordinate systems exist in astronomy? What are they? Describe in which context each one is useful.

   - The two most used coordinate systems are the altazimuth coordinate system and the equatorial coordinate syatem. The altazimuth system is useful for specifying the direction of an object when you are standing at a point on the Earth. However, it is not very useful for uniquely specifying the coordinates of an object when communicating with someone, because the altazimuth coordinates of a given object vary from place to place and from time to time. For this, we use the equatorial coordinate system, which is fixed to the Earth.

4. Convert the following decimal coordinates RA: 83.63, DEC: 22.014 into sexagesimal coordinates (hour, minutes, seconds for RA and degrees, minutes, seconds for Dec). Show your calculation. These are the coordinates of a famous astronomical object. Please look up what this object is, and describe it.

   - First, the RA. Since there are 15 degrees in 1 hour of RA, we can write:

   $$83.63^\circ = \frac{83.63^\circ}{15\frac{^\circ}{\mathrm{hr}}} = 5.575\ \mathrm{hr}$$

   Now each hour is made up of 60 minutes, and each minute is made up of 60 seconds, so 0.575 hr = 34.5 m = 34 m 30 s. So:

   $$83.63^\circ\ \ \mathrm{RA} = 5\ \mathrm{h}\ 34\ \mathrm{m}\ 30\ \mathrm{s}.$$

- Now the declination. The 22° remains unchanged, and we can write

$$.014^\circ = .014^\circ \times 60\frac{'}{^\circ} = 0.84' = 0.84' \times 60\frac{''}{'} = 50.4''$$

So:

$$22.014^\circ \ \ \mathrm{Dec} = 22^\circ\ 0'\ 50.4''$$

- These are the coordinates of the Crab Nebula, also known as Messier 1. It is the remnant of a supernova that exploded in 1054 AD.

5. What is the equivalent of the Equator on earth called on the Celestial Sphere?

   - It is called the Celestial Equator.

6. You are at the Palomar observatory at the Hale telescope, a 5 meter optical telescope in California. The coordinates of Palomar on earth are $33.36^\circ\mathrm{N}, 116.86^\circ\mathrm{W}$. What is the declination of the Zenith where you are?

   - The declination of the zenith is always equal to your latitude. Draw a picture of the Earth with you standing on it and you will see this to be true. So the declination of the zenith at Mt Palomar is equal to 33.36°.

7. How many sidereal days are there in a solar year? Explain your answer.

   - There is one more sidereal day in a year than there are solar days, so there are 366 (366.2425 to be more exact) sidereal days in a year.

8. We said that you can only see one half of the sky at any one time on the surface of the Earth. However, because you are above the ground and the Earth is curved, you can actually see slightly below the theoretical horizon. If your eyes are 2 meters off the ground, and the Earth is a perfect sphere with a radius of 6400 km, how far below the theoretical horizon can you see? How about if you are at the top of the One World Trade Center, 1776 feet (541 meters) off the ground? Give both answers in minutes of arc.

   - Refer to the figure below. You are standing at point C, a distance h above the surface of the Earth, which has radius R. Because you are above the surface, you can see a distance θ below the theoretical horizon, which is shown by the horizontal line in the figure. This is the same angle as angle CAB, since angle BCA is equal to $90 - \theta$. Now look at triangle ABC. One side has length R, and the hypotenuse has length R+h, so the other side (BC) has length $\sqrt{(\mathrm{R}+\mathrm{h})^2 - \mathrm{R}^2}$ Since h is much much less than R, we can neglect h^2 in comparison to Rh, and we can write:

$$\mathrm{BC} = \sqrt{(\mathrm{R}+\mathrm{h})^2 - \mathrm{R}^2} = \sqrt{\mathrm{R}^2 + 2\mathrm{Rh} + \mathrm{h}^2 - \mathrm{R}^2} \approx \sqrt{2\mathrm{Rh}}$$

Now the angle $\theta = \arctan(\mathrm{BC}/\mathrm{AB})$, and since this angle is very small, we can use the small angle approximation and write:

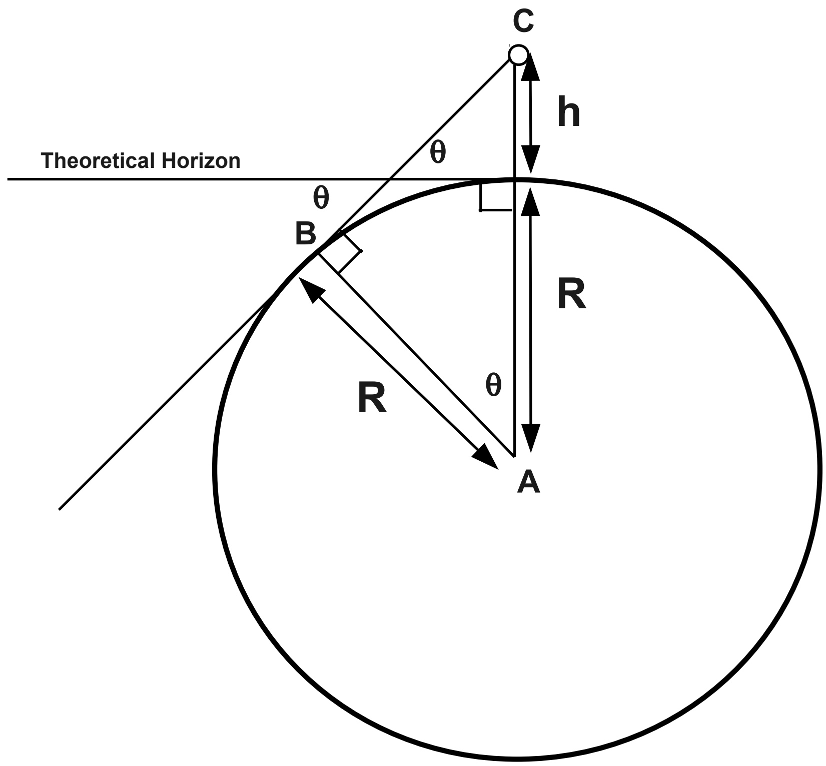
$$\theta \approx \frac{\mathrm{BC}}{\mathrm{AB}} = \frac{\sqrt{2\mathrm{Rh}}}{\mathrm{R}} = \sqrt{\frac{2\mathrm{h}}{\mathrm{R}}}$$

Now we put in numbers. R is 6400 km or 6.4×10^6 m, and h is 2 m in the first case and 541 m in the second case. So, for the first case:

$$\theta = \sqrt{\frac{2\mathrm{h}}{\mathrm{R}}} = \sqrt{\frac{2 \times 2\ \mathrm{m}}{6.4 \times 10^6\ \mathrm{m}}} = 7.9 \times 10^{-4}\ \mathrm{radians} = 0.045^\circ = 2.7'$$

For the second case:

$$\theta = \sqrt{\frac{2\mathrm{h}}{\mathrm{R}}} = \sqrt{\frac{2 \times 541\ \mathrm{m}}{6.4 \times 10^6\ \mathrm{m}}} = .013\ \mathrm{radians} = 0.74^\circ = 44'$$


C
h
Theoretical Horizon
θ
θ
B
R
R
θ
A