Observational Astronomy - Spring 2014
Homework 10 - Cosmology I

1. Figure 1 on the next page shows the Period-Luminosity relation for Cepheid variable stars, and Figure 2 shows measurements of the brightness of a Cepheid variable in the Andromeda galaxy. Using these two figures, calculate the distance to the Andromeda galaxy. Use the average brightness of the Andromeda Cepheid when applying to Figure 1.

   - The Cepheid in Andromeda has a period of 31.4 days, and an average apparent magnitude of about 18.7. Reading off of Figure 1, we see that a Cepheid with a period of 31.4 days has an absolute magnitude of about -5.5. Now apply the relation between apparent and absolute magnitude:

$$\begin{aligned} \mathrm{m} &= \mathrm{M} - 5 + 5\log_{10}(\mathrm{D}) \\ 18.7 &= -5.5 - 5 + 5\log_{10}(\mathrm{D}) \\ \log_{10}(\mathrm{D}) &= \frac{29.2}{5.0} = 5.84 \\ \mathrm{D} &= 10^{5.84} = 692\,\mathrm{kpc} \end{aligned}$$

   - This is reasonably close to the accepted distance of 780 kpc.

2. The H_β spectral line is measured in the lab to have a wavelength of 486.1 nm. You measure the spectrum of a nearby star and find that this line is blue-shifted and has a wavelength of 485.7 nm. Is the star approaching us or receding from us? At what velocity?

   - Since the wavelength is blueshifted, the star is approaching us. The velocity of approach is given by:

$$\frac{\mathrm{v}}{\mathrm{c}} = \mathrm{z} = \frac{\lambda_{\mathrm{observed}} - \lambda_{\mathrm{emitted}}}{\lambda_{\mathrm{emitted}}} = \frac{485.7 - 486.1}{486.1} = -0.00082$$

   - The minus sign tells us that it is approaching. The velocity of approach is given by:

$$\mathrm{v} = \mathrm{c} * 0.00082 = 3.0 \times 10^5\,\frac{\mathrm{km}}{\mathrm{sec}} \times 0.00082 = 247\,\frac{\mathrm{km}}{\mathrm{sec}}$$

3. You measure the spectrum of a galaxy and find that the galaxy has a redshift z = 0.06. How fast is it moving away from us? If the Hubble constant is 70 km/sec/Mpc, how far away is the galaxy?

   - With a redshift of z = 0.06, the galaxy is receding at:

$$\mathrm{v} = \mathrm{c} * 0.06 = 3.0 \times 10^5\,\frac{\mathrm{km}}{\mathrm{sec}} \times 0.06 = 18,000\,\frac{\mathrm{km}}{\mathrm{sec}}$$

   - Using the Hubble constant of 70 km/sec/Mpc gives a distance of

$$\mathrm{D} = \frac{18,000\,\frac{\mathrm{km}}{\mathrm{sec}}}{70\,\frac{\mathrm{km}}{\mathrm{sec\,Mpc}}} = 257\,\mathrm{Mpc}$$

4. Let's estimate the age of the universe. Pick two galaxies, one 10 Mpc away, and one 100 Mpc away. Calculate how fast each one is moving away from us, assuming a Hubble constant of 70 km/sec/Mpc. Assuming they have always been moving at that speed, calculate how long it has taken them to reach their present distance. Do you see that this will give the same result no matter how far away the galaxy is? What is your estimate of the age of the Universe given this procedure?

   - The speed of recession is just the distance times the Hubble constant:

$$\mathrm{D}_{10} = 10\,\mathrm{Mpc} \times 70 \frac{\mathrm{km}}{\mathrm{sec}\,\mathrm{Mpc}} = 700\,\mathrm{km/sec}$$

$$\mathrm{D}_{100} = 100\,\mathrm{Mpc} \times 70 \frac{\mathrm{km}}{\mathrm{sec}\,\mathrm{Mpc}} = 7,000\,\mathrm{km/sec}$$

   - Now we know that distance = velocity x time, so the time to reach their present distance is just their distance divided by their velocity:

$$\mathrm{T}_{10} = \frac{10\,\mathrm{Mpc} \times 3.1 \times 10^{19}\,\frac{\mathrm{km}}{\mathrm{Mpc}}}{700\,\frac{\mathrm{km}}{\mathrm{sec}}} = 4.4 \times 10^{17}\,\mathrm{sec}$$

$$\mathrm{T}_{100} = \frac{100\,\mathrm{Mpc} \times 3.1 \times 10^{19}\,\frac{\mathrm{km}}{\mathrm{Mpc}}}{7000\,\frac{\mathrm{km}}{\mathrm{sec}}} = 4.4 \times 10^{17}\,\mathrm{sec}$$

   - Note that since the velocity is proportional to the distance, all distances give the same answer. Converting this to years gives:

$$\frac{4.4 \times 10^{17}\,\mathrm{sec}}{3600\frac{\mathrm{sec}}{\mathrm{hr}}24\frac{\mathrm{hr}}{\mathrm{day}}365.25\frac{\mathrm{days}}{\mathrm{yr}}} = 14 \times 10^{9}\,\mathrm{years}$$

   - This is just an estimate, but it is close to the accepted age of 13.7 billion years