Celestial Coordinate Systems

Craig Lage
Department of Physics, New York University, csl336@nyu.edu

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1 Introduction

This document reviews briefly some of the key ideas that you will need to understand in order to identify and locate objects in the sky. It is intended to serve as a reference document.

2 Angular Basics

When we view objects in the sky, distance is difficult to determine, and generally we can only indicate their direction. For this reason, angles are critical in astronomy, and we use angular measures to locate objects and define the distance between objects. Angles are measured in a number of different ways in astronomy, and you need to become familiar with the different notations and comfortable converting between them. A basic angle is shown in Figure 1.

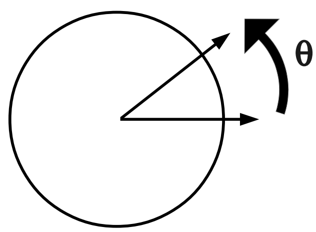


Figure 1: A basic angle, θ.

We review some angle basics. We normally use two primary measures of angles, degrees and radians. In astronomy, we also sometimes use time as a measure of angles, as we will discuss later. A radian is a dimensionless measure equal to the length of the circular arc enclosed by the angle divided by the radius of the circle. A full circle is thus equal to 2π radians. A degree is an arbitrary measure, where a full circle is defined to be equal to 360°. When using degrees, we also have two different conventions, to divide one degree into decimal degrees, or alternatively to divide it into 60 minutes, each of which is divided into 60 seconds. These are also referred to as minutes of arc or seconds of arc so as not to confuse them with minutes of time and seconds of time. Specifying an angle in degrees, minutes, and seconds is referred to as *sexagesimal* notation. You should be familiar with the following relations:

$$\text{Full Circle} = 360 \text{ degrees} = 360^\circ = 2\pi \text{ radians}$$

$$1^\circ = 60 \text{ minutes of arc} = 60'$$

$$1 \text{ minute of arc} = 60 \text{ seconds of arc} = 60''$$

3 Locations on the Earth - Latitude and Longitude

To specify a direction in space from a given point requires two angles. To specify a location on the surface of the Earth, we use two angles as measured from the center of the Earth, referred to as latitude and longitude. For each of the two angles we need a reference point to measure from. Since the Earth is spinning, we have a natural reference point to use in one direction. We use the equator as the zero point of latitude, so that the equator has latitude of zero degrees, and north and south poles have latitudes of $+90°$ and $+90°$, respectively. In the longitude direction, there is no such natural reference point, so we need to arbitrarily identify a point to be zero longitude. For many years, each country had their own zero longitude standard, but today, the Royal Observatory in Greenwich, England is the internationally recognized standard for zero longitude. The zero longitude line is known as the *prime meridian*. Figure 2 shows these ideas schematically.

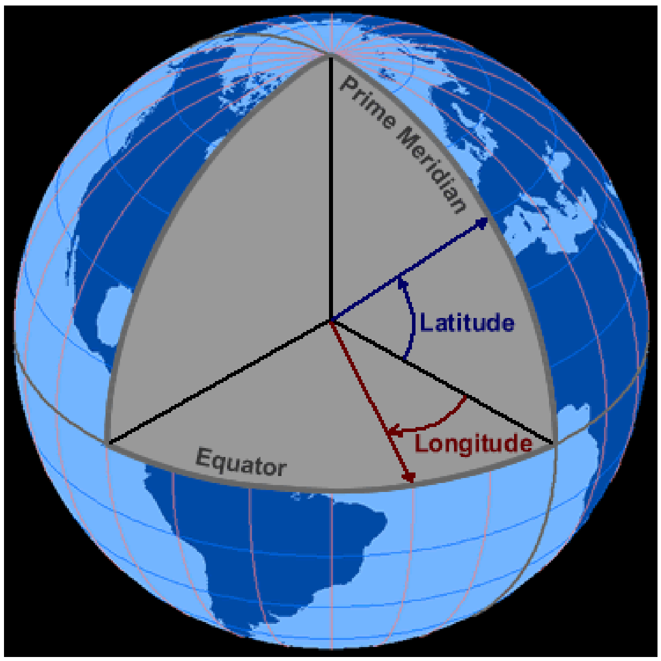


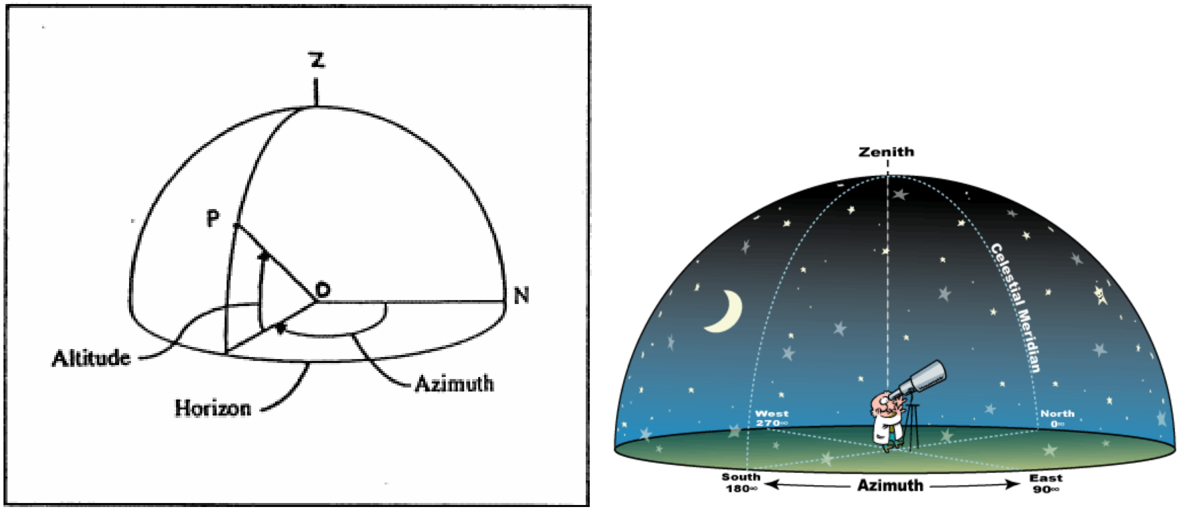
Figure 2: Latitude and Longitude are used to measure location on the surface of the Earth.

4 Locations on the Sky - Altitude and Azimuth

A location on the sky is also specified by a direction in space and so also requires two angles. The simplest scheme, referred to as *altazimuth* coordinates, is to use the horizon as the reference for one angle. A second angle requires a reference point, which we arbitrarily designate as the direction North. The terminology here is as follows:

- The angle above the horizon is called the *altitude*
- The angle measured from North is called the *azimuth*
- The line on the sky from due North to due South is called the *meridian*
- An object crossing the meridian is said to be *transiting*
- The point straight up (altitude $= 90°$) is called the *zenith*
- The point straight down (altitude $= -90°$) is called the *nadir*

Figure 3 shows these ideas schematically.

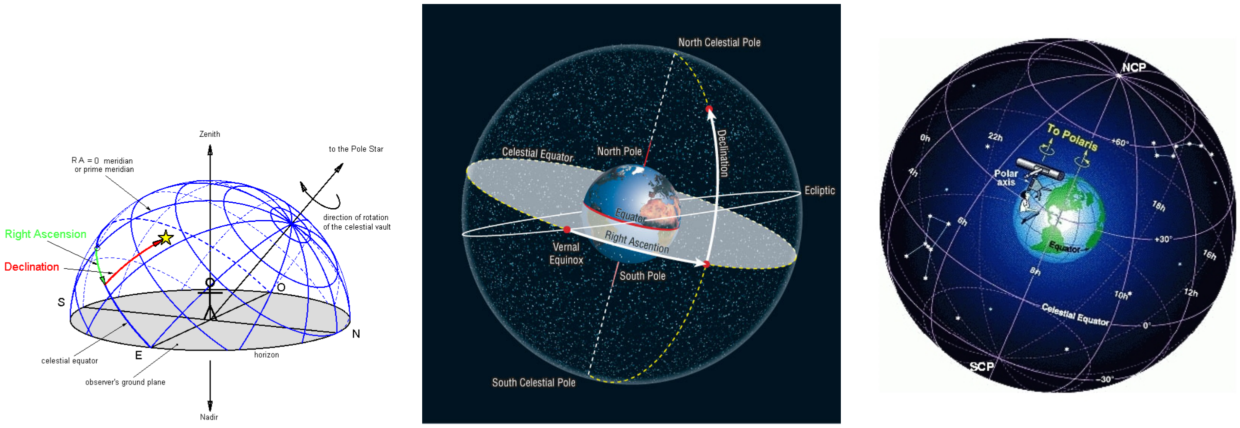


(a) (b)

Figure 3: Altazimuth coordinates

5 Locations on the Sky - Declination and Right Ascension

Altazimuth coordinates are simple and convenient, but they have major drawbacks. First, the altazimuth coordinates of a celestial object will be different when measured from different locations on the surface of the Earth. Second, even from a fixed location, the altazimuth coordinates of an object will vary in time as the object rises and sets. Clearly, to specify an object in the sky, we would like a set of coordinates which is the same for everyone, and which does not change with time. We do this using what are called equatorial coordinates. These result from taking the latitude and longitude coordinates that we use on the Earth, and projecting them out onto the sky. Projecting the Earth's equator onto the sky gives us the *celestial equator*. Lines of latitude can also be projected onto the sky. When used as celestial coordinates, the analogue of latitude is referred to as *declination*. An object on the celestial equator has a declination of $0°$, an object at the north celestial pole has a declination of $+90°$, and an object at the south celestial pole has a declination of $-90°$. In celestial coordinates, the analogue of longitude is referred to as *right ascension*. Note that, just like on the Earth, we need to specify an arbitrary zero point to measure right ascension from. Because the Earth is rotating, we cannot directly translate from a point on the Earth. So we specify the location of a given celestial object. The location of the sun as it crosses the equator on the vernal equinox (the first day of spring) is designated as the zero point of right ascension. Another way of saying the same thing is that the zero point of right ascension is the intersection point between the celestial equator with the plane of the Earth's orbit. The plane of the Earth's orbit, projected onto the sky, is also the path followed by the sun across the sky. This path is referred to as the *ecliptic*. Figure 4 shows these ideas schematically.



(a) (b) (c)

Figure 4: Equatorial coordinates

5.1 Measurement of Right Ascension

Although we sometimes measure right ascension in degrees, it is more usual to measure right ascension using time as a variable. To do this, instead of dividing the circle up into 360°, we divide it up into 24 hours, with each hour divided up into 60 minutes, and each minute divided up into 60 seconds. A common point of confusion is to confuse minutes of right ascension with minutes of arc, and seconds of right ascension with seconds of arc. Since these are different units, they are not the same. To help keep them straight, we use different symbols for them, as follows. A typical right ascension of 3 hours, 24 minutes, 13 seconds is written as: $3^{\mathrm{h}}24^{\mathrm{m}}13^{\mathrm{s}}$, while a typical declination measurement of 3 degrees, 24 minutes, 13 seconds is written as: $3°24'13''$. Table 1 gives some useful conversions. There are also many online converters available.

Parameter	Value in ° ′ “	Value in Decimal °	Value in $^{\mathrm{hms}}$
Full Circle	$360°0'0''$	360.0°	$24^{\mathrm{h}}0^{\mathrm{m}}0^{\mathrm{s}}$
1^h	$15°0'0''$	15.0°	$1^{\mathrm{h}}0^{\mathrm{m}}0^{\mathrm{s}}$
1^m	$0°15'0''$	0.25°	$0^{\mathrm{h}}1^{\mathrm{m}}0^{\mathrm{s}}$
1^s	$0°0'15''$	0.004167°	$0^{\mathrm{h}}0^{\mathrm{m}}1^{\mathrm{s}}$
$1°0'0''$	$1°0'0''$	1.0°	$0^{\mathrm{h}}4^{\mathrm{m}}0^{\mathrm{s}}$
$0°1'0''$	$0°1'0''$	0.0167°	$0^{\mathrm{h}}0^{\mathrm{m}}4^{\mathrm{s}}$
$0°0'1''$	$0°0'1''$	0.000277°	$0^{\mathrm{h}}0^{\mathrm{m}}0.0667^{\mathrm{s}}$

Table 1: Conversions between angular measures

6 Sidereal Time and Solar Time

Because the direction to objects in the sky is constantly changing due to the rotation and orbital motion of the Earth, time is crucial in astronomy. This section reviews some astronomical time measures. The time between when the sun has the same azimuth on successive days (for example, the time from noon on one day until noon on the next day) is referred to as a *solar day*. The time between when a distant star has the same azimuth on successive days (for example, when the star transits on one day until it transits again on the next day) is referred to as a *sidereal day*. The reason why these two days are different is shown in Figure 5. After one sidereal day, the Earth has completed one full rotation. However, during the time that it has taken to complete this rotation, it has moved on slightly in its orbit about the sun, so it has to rotate a little bit further to bring the sun back to the same apparent location. Therefore, a solar day is slightly longer than a sidereal day. This difference is about $3^{\mathrm{m}}56^{\mathrm{s}}$. Another way of seeing this is that, because the Earth is orbiting around the sun, while there are

365 (actually 365.24219042) solar days in a year, there are 366 (actually 366.24219042) sidereal days in a year.

What does this mean practically in astronomy? Our clock time is based on the sun, because we want the day and night cycle to be synchronized with our clocks. This means that if we track a given star, the star will transit $3^{\mathrm{m}}56^{\mathrm{s}}$ earlier each night. If we want to track astronomical objects, it is useful to have clocks which track *sidereal time*. Sidereal time divides the sidereal day up into 24 hours, so sidereal time runs slightly faster than normal(solar) time. It is very useful to know the *local sidereal time* (LST) at your location. You can calculate this from the ideas described here, or there are converters available on the internet. This brings us to the important concept of *hour angle*, which is the angular distance of an object away from the meridian, usually measured in time units. Thus an object which is transiting has an hour angle of 0, and object to the east of the meridian has a negative hour angle, and an object to the west of the meridian has a positive hour angle. There is an important relation written as follows:

$$\text{Hour Angle} = \text{Local Sidereal Time} - \text{Right Ascension} \tag{1}$$

This means that the objects currently transiting have a right ascension equal to your local sidereal time. Objects to the east of your meridian have a right ascension greater than your local sidereal time, and objects to the west of your meridian have a right ascension less than your local sidereal time. This should make it clear why we find it convenient to specify right ascension using time as a measure.

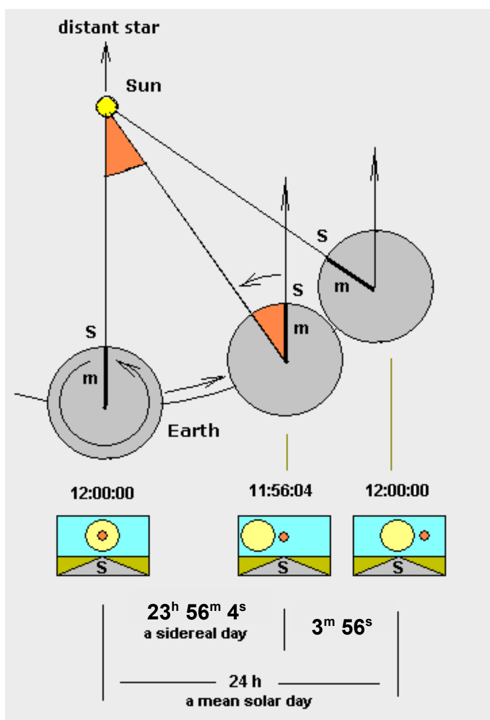


Figure 5: The reason why the solar day and the sidereal day differ.

7 Angular Size and Distance

In general, distances in astronomy are difficult to determine, and we often know the position of an object on the sky without knowing its distance. However, there is a relation between an object's angular size and its distance. This means that if we do know the distance to an object, we can calculate its actual size from its angular size, or conversely if we know it actual size, we can calculate its distance from its angular size. Referring to Figure 6, simple trigonometry gives us the following relation:

$$\tan(\theta) = \frac{\mathrm{R}}{\mathrm{D}}. \tag{2}$$

Usually in astronomy the angular size of an object is quite small. For small angles, the tangent of the angle is approximately equal to the angle itself measured in radians, so we can write:

$$\theta = \frac{\mathrm{R}}{\mathrm{D}}. \tag{3}$$

This equation assumes that the angle θ is measured in radians.

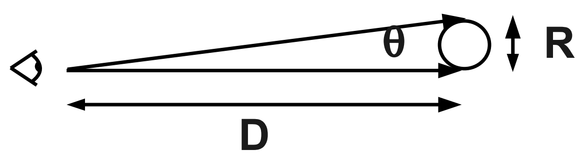


Figure 6: The relation between the angular size of an object, its distance, and its actual size.