Measurements and Simulations of the Brighter-Fatter Effect

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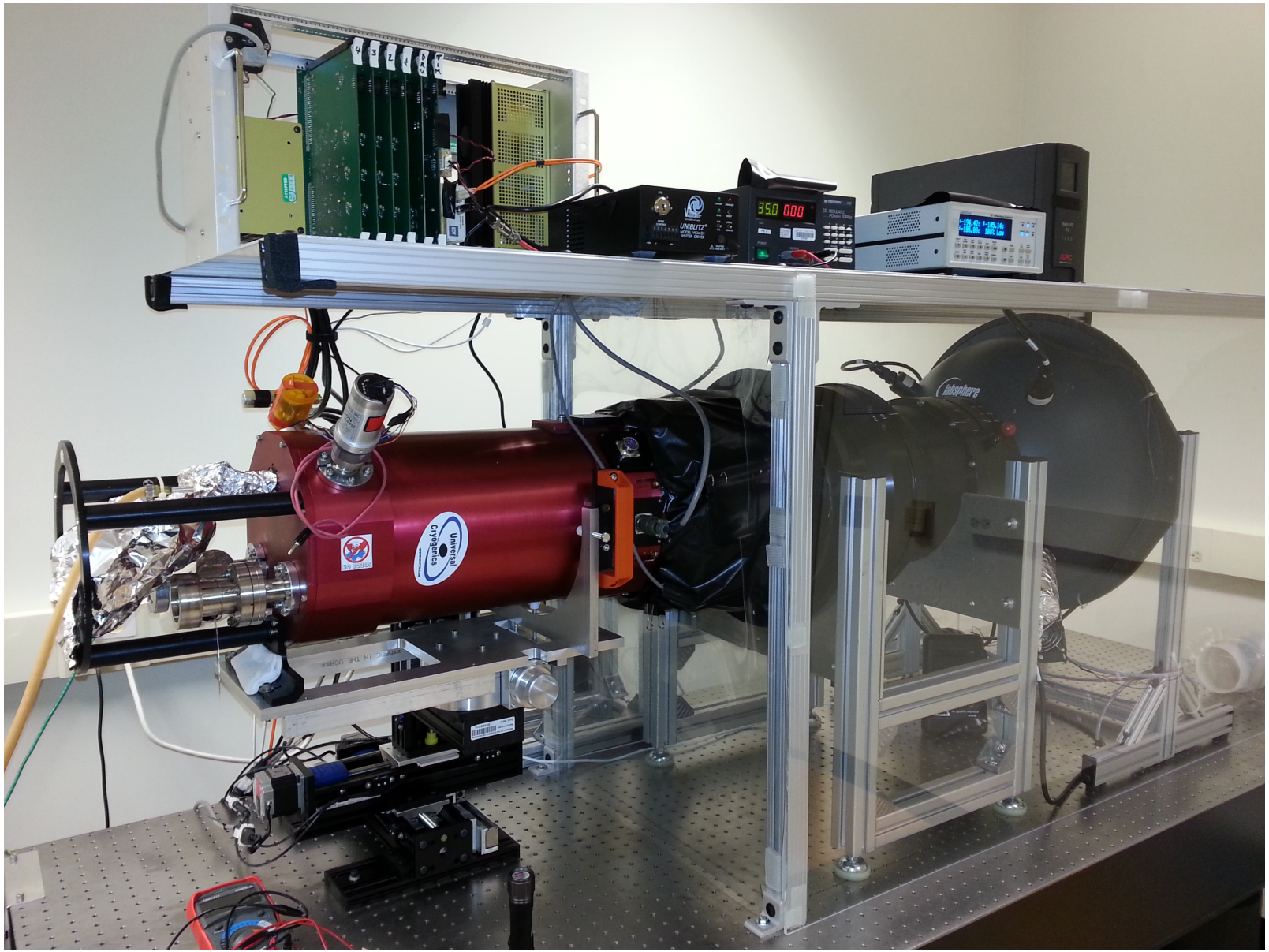
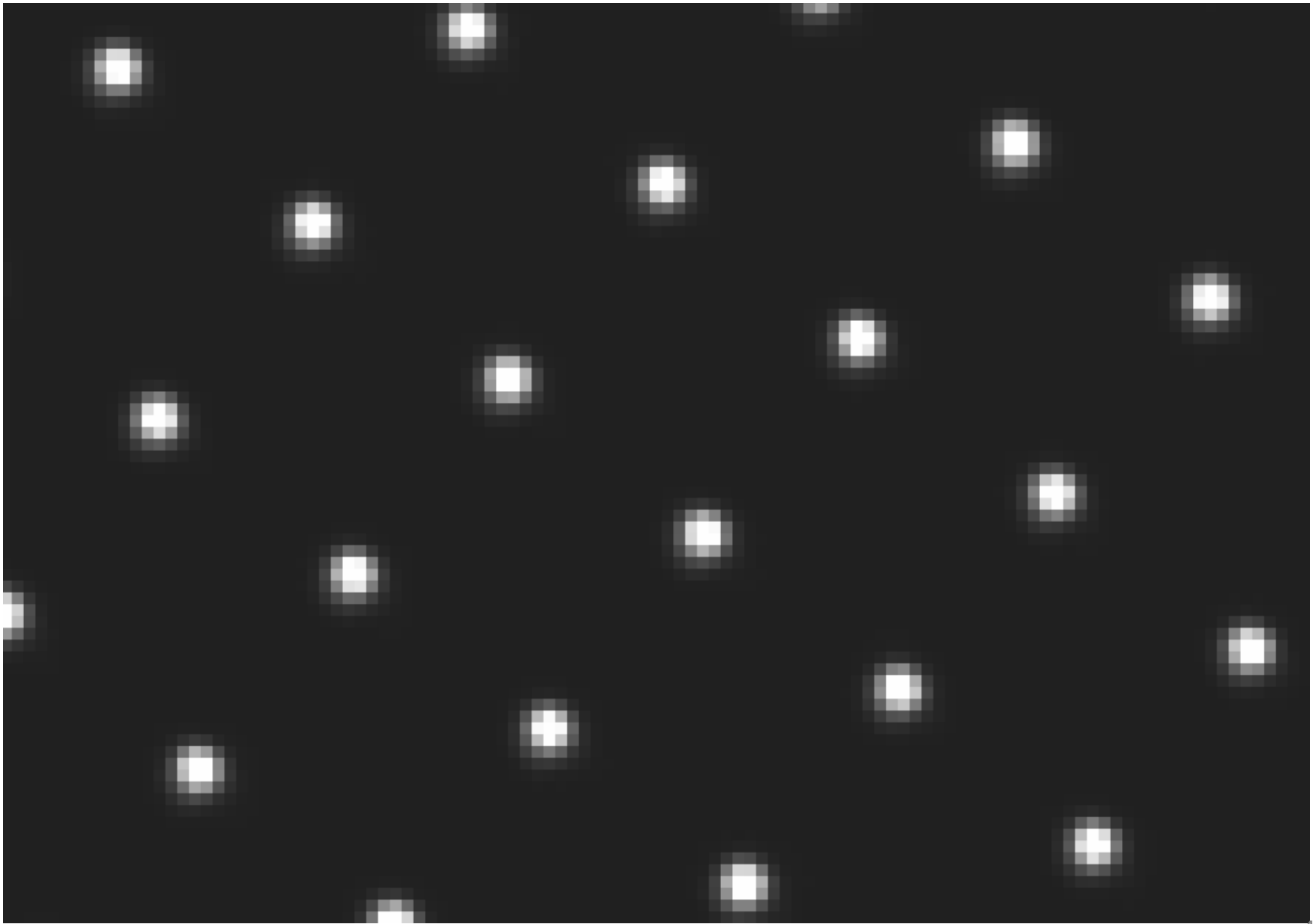
Outline

- Introduction
- Forward Modeling of Spot images to measure B-F effect.
- Simulations of B-F effect.
- Measurement - Simulation comparison.
- Modeling Stored Charge Self-Consistently
- What are the Free Parameters?
- How to Model Saturation Effects?
- Conclusions and Next Steps

LSST Optical Simulator



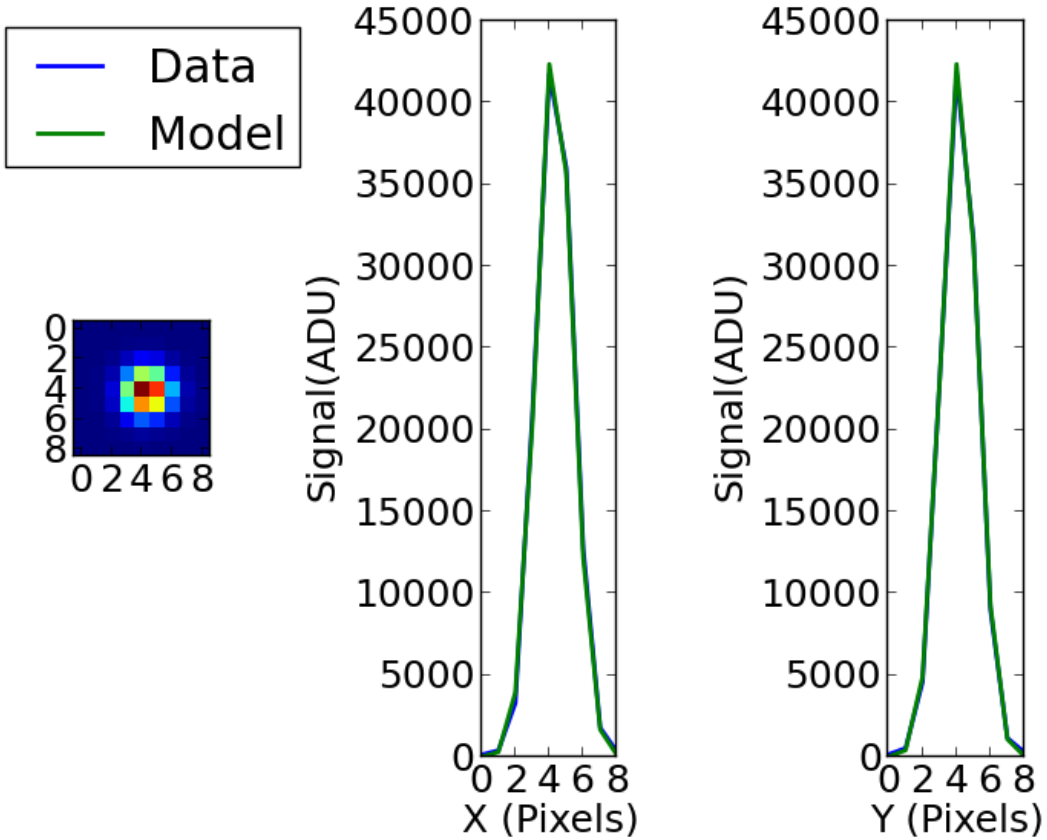
Typical Image of 30 micron Spots



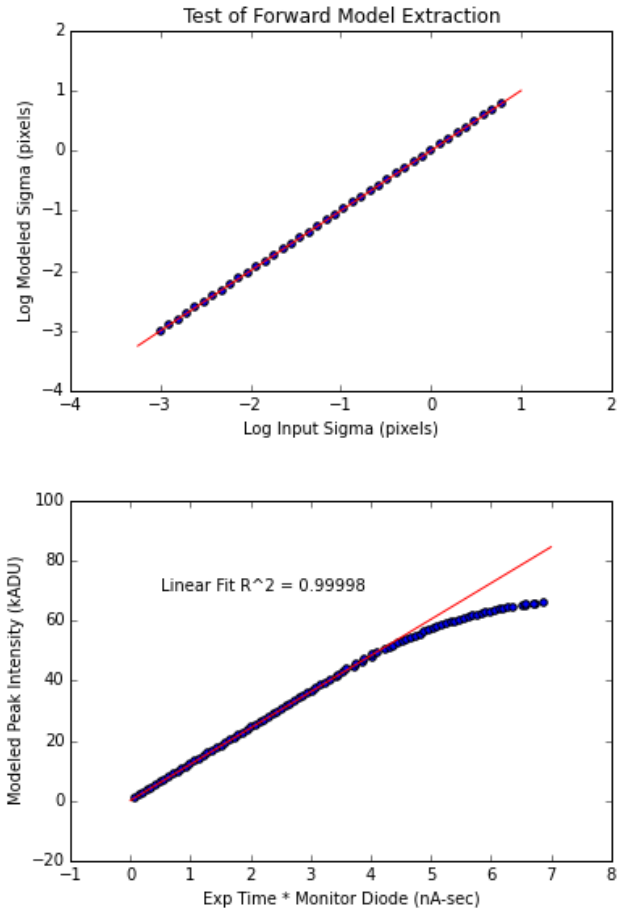
Forward modeling of Discrete Spots

- Use sextractor to identify a list of spots.
  - Typically 1000-2000 in one CCD segment depending on size of window.
  - Use sextractor central pixel location, but not size or exact offset within pixel.
  - Use a constant window ("postage stamp") for all spots. Using 9x9 pixels.
- Assume all spots have the same shape, but allow variable peak intensity and offset within central spot.
  - Calculate first moment of postage stamp to determine offset within central pixel.
- Assume a 2D Gaussian, calculate expected signal in each pixel
  - $\mathrm{I} = \mathrm{I}_0(\mathrm{erf}(\frac{\mathrm{x_{max}}}{\sqrt{2}\sigma_\mathrm{x}}) - \mathrm{erf}(\frac{\mathrm{x_{min}}}{\sqrt{2}\sigma_\mathrm{x}})) * (\mathrm{erf}(\frac{\mathrm{y_{max}}}{\sqrt{2}\sigma_\mathrm{y}}) - \mathrm{erf}(\frac{\mathrm{y_{min}}}{\sqrt{2}\sigma_\mathrm{y}}))$
  - Find $(\sigma_\mathrm{x}, \sigma_\mathrm{x})$ which minimizes: $\sum_{\mathrm{Nspots}} \sum_{\mathrm{x,y}} (\mathrm{Measured_{n,x,y}} - \mathrm{Calculated_{n,x,y}})^2$

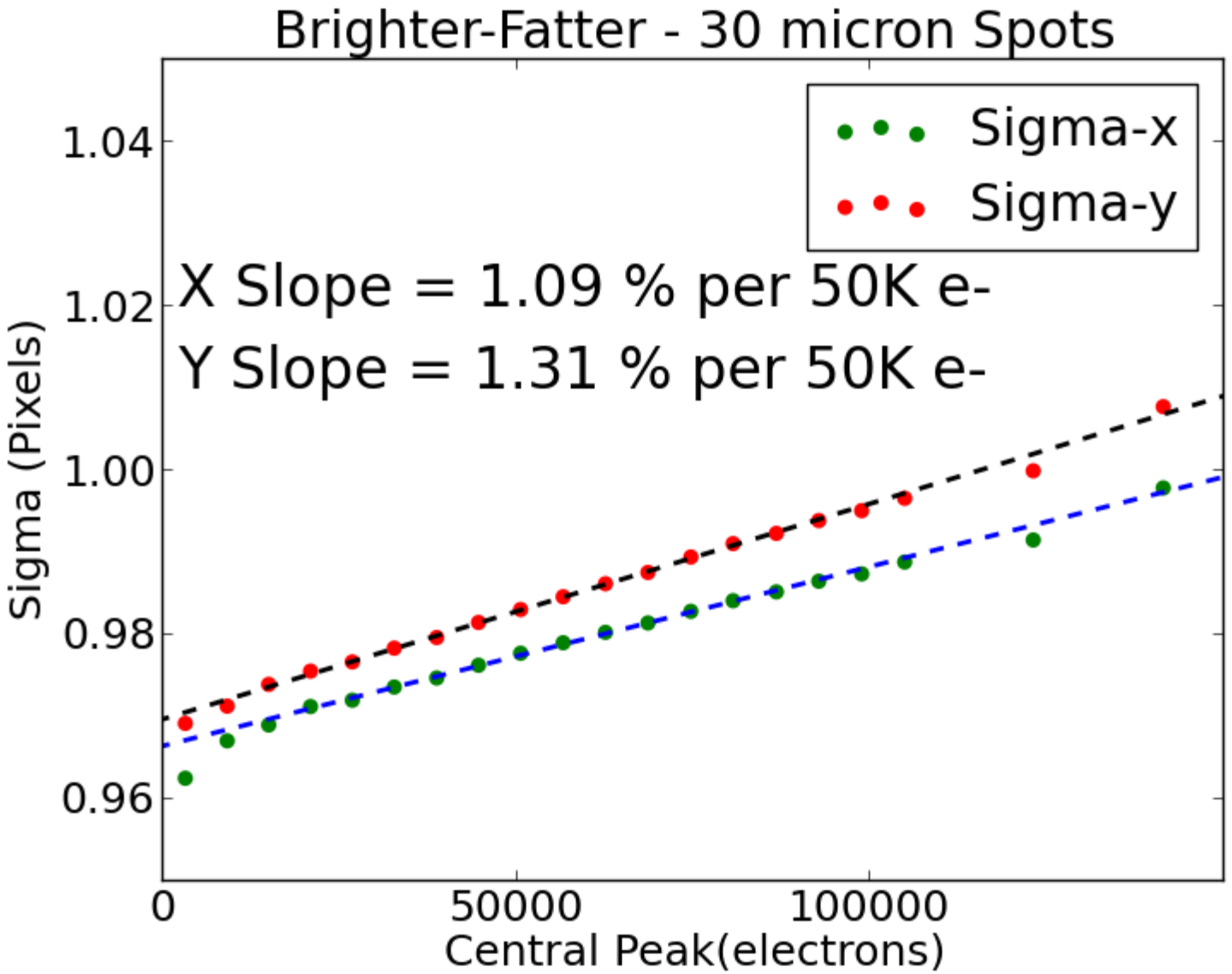
Checks on Forward modeling



Vertical Scale in ADU.
Multiply by 2.5 for electrons.



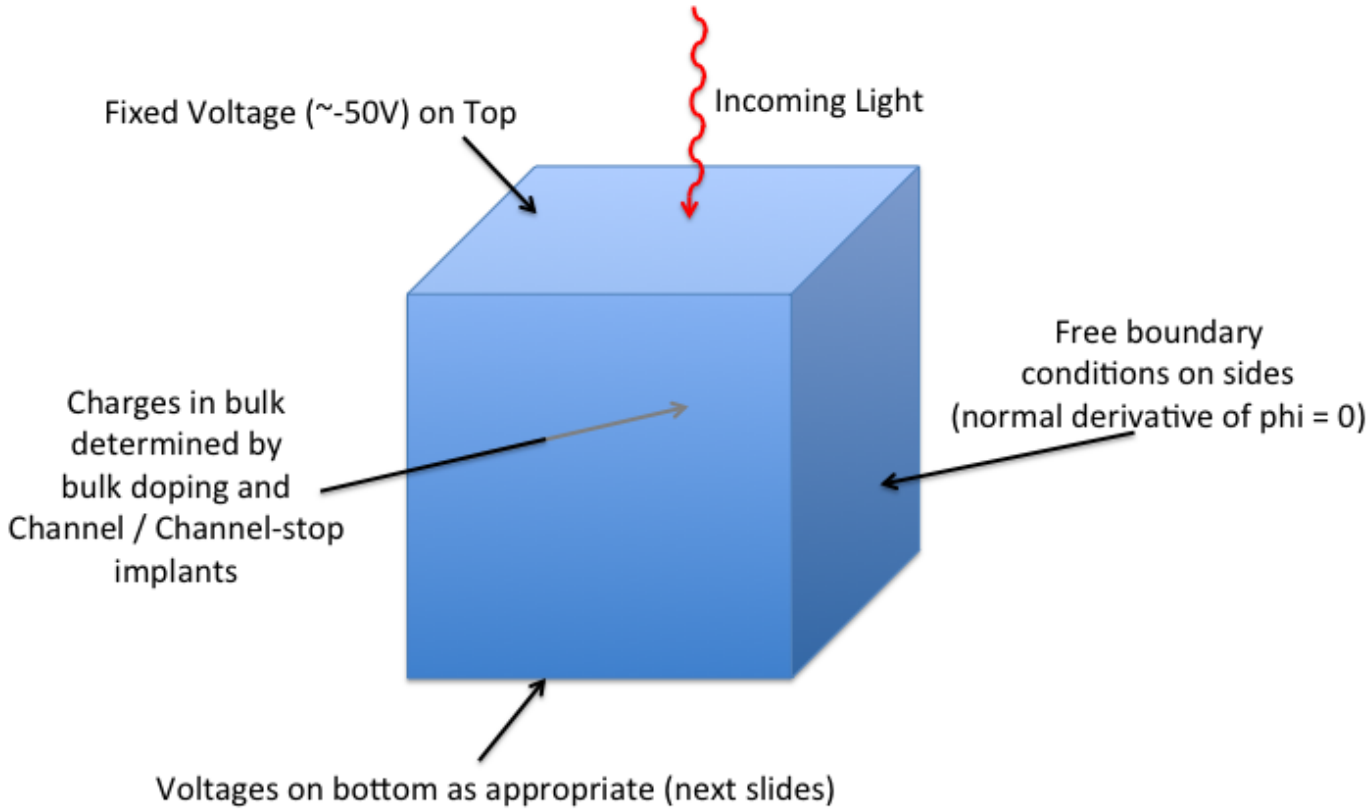
Initial B-F measurements



Simulations of B-F effect.

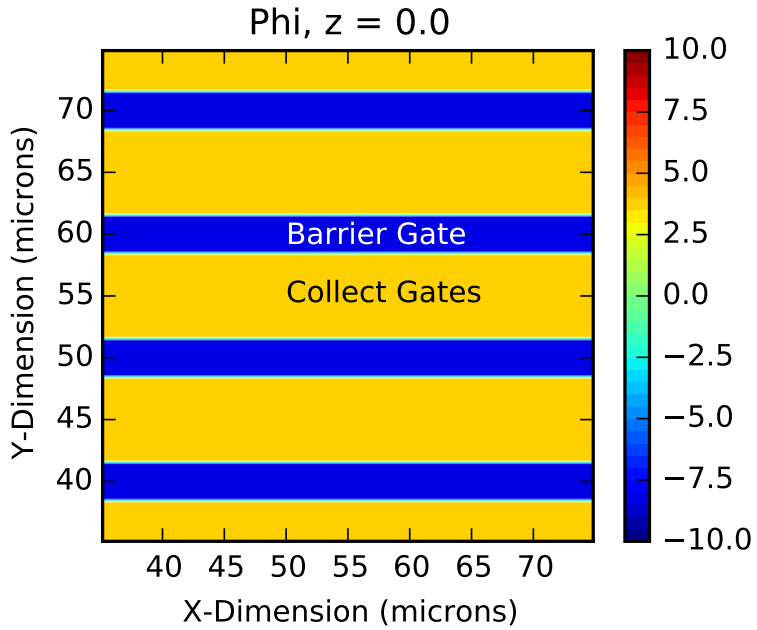
- Brief review of Poisson Solver.
- Diffusion model
- Addition of code to simulate B-F effect.

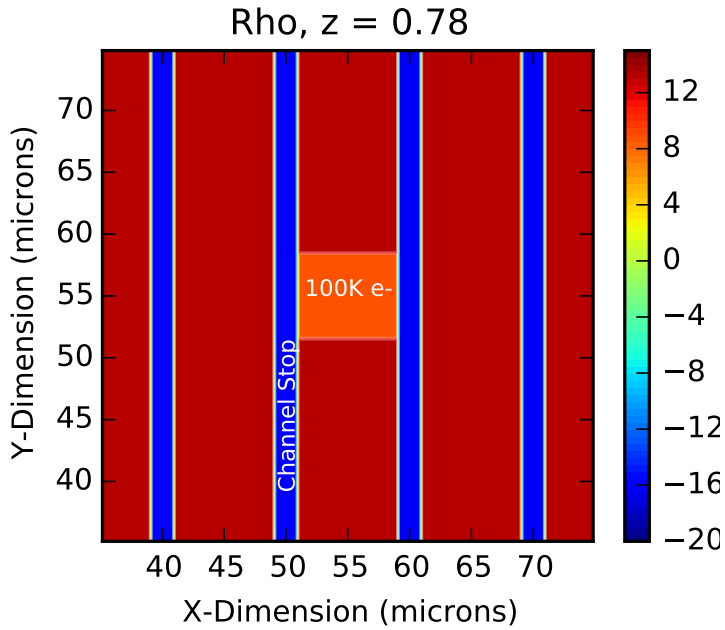
Typical Simulation $100\mu\mathrm{m}$ Cube.

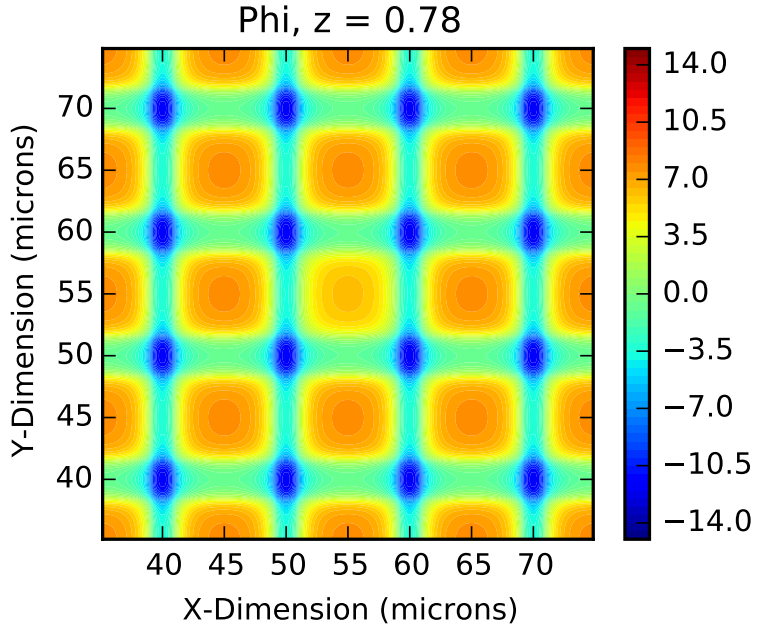


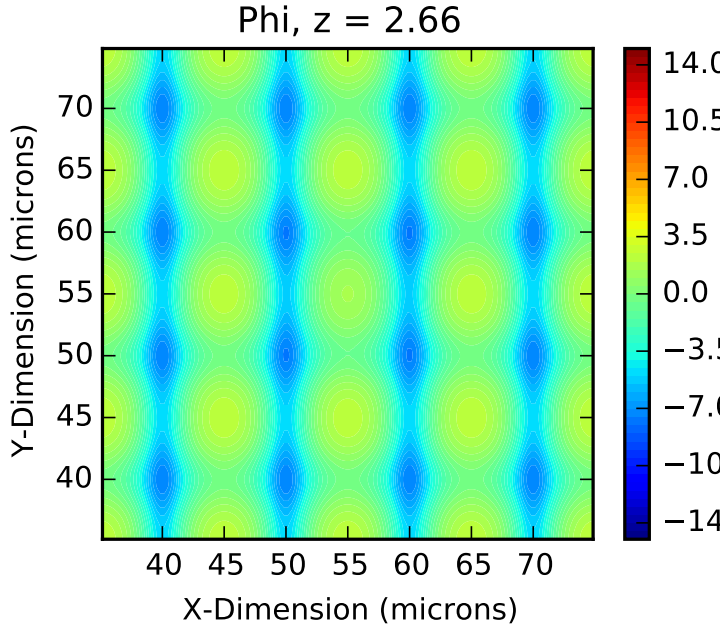
- $100\mu\mathrm{m}$ Cube. - 10 X 10 pixels in X and Y.
- 32 grid cells per pixel - cell size $= 0.31\ \mu$.

Pixel Array Summary Plot



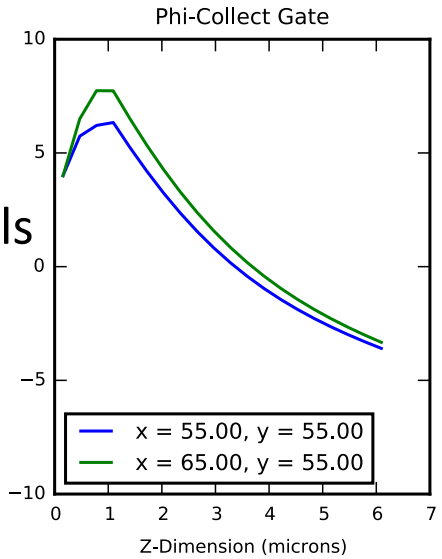


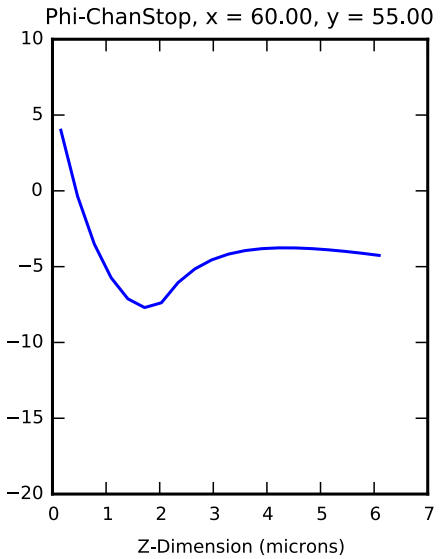


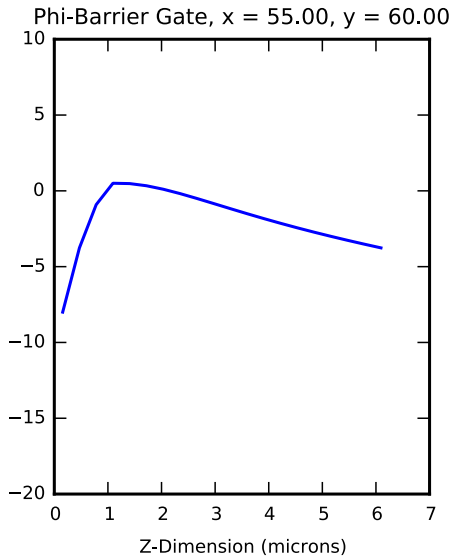


Potentials and Charges - Z-slice at Pixel Center

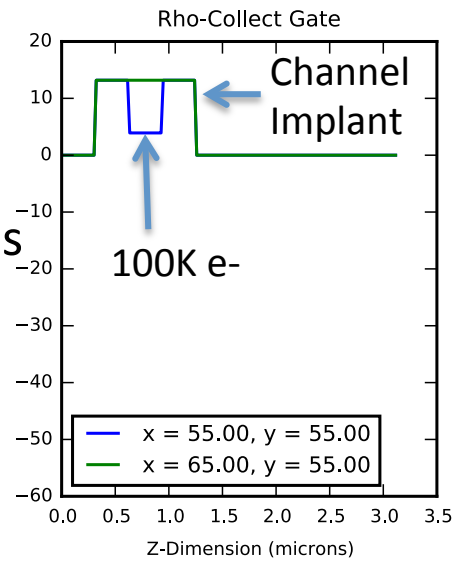
Potentials

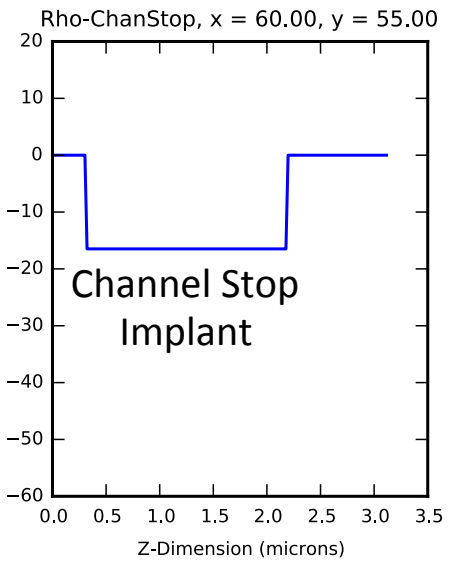


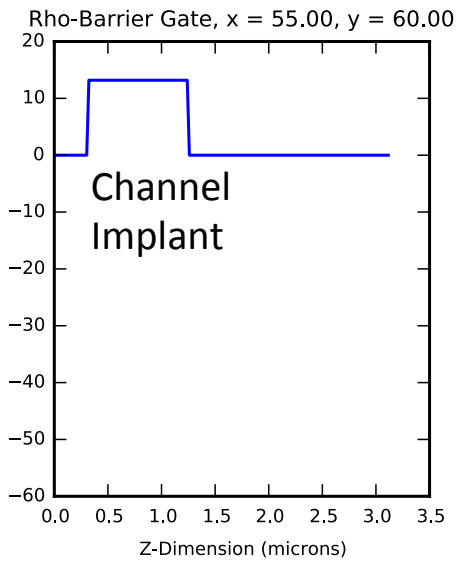




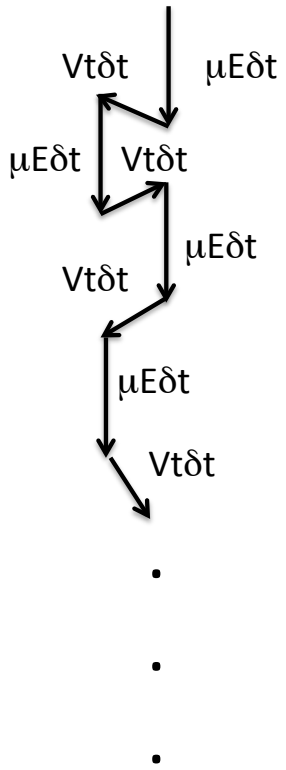
Charges





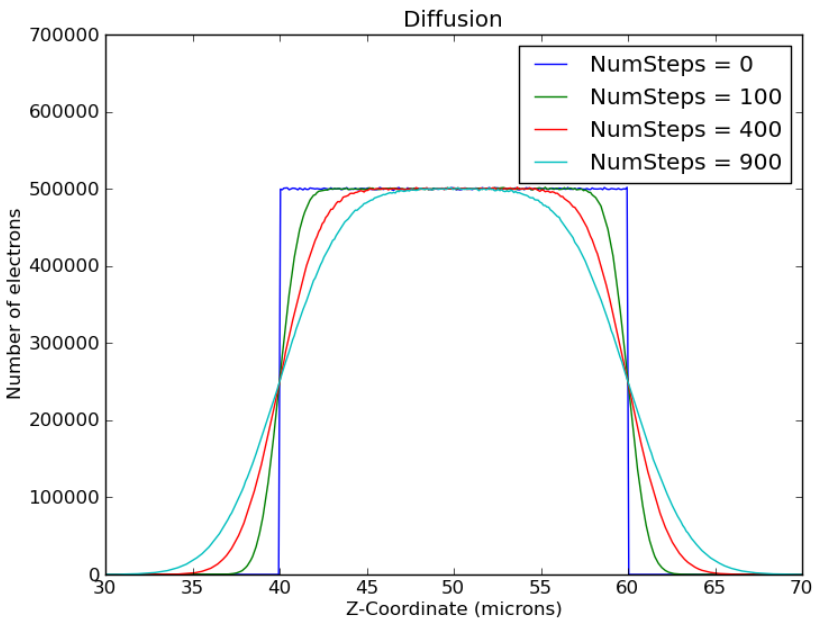


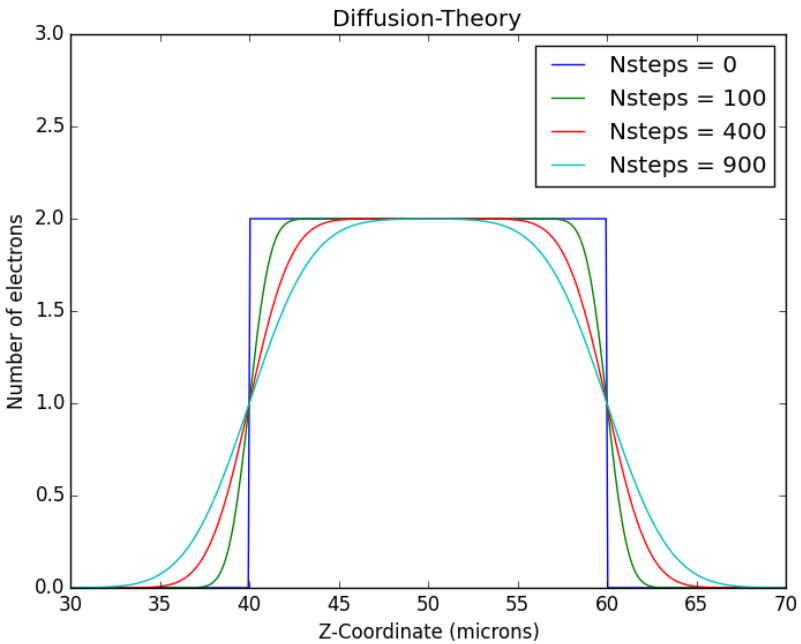
Diffusion Model



- Mobility: $\mu(E, T)$ calculated from Jacobini model
- $\mu = 1584 \frac{\mathrm{cm}^2}{\mathrm{V-sec}}$ at $\mathrm{E} = 6000 \frac{\mathrm{V}}{\mathrm{cm}}$
- Collision time:
- $\tau = \frac{\mathrm{m}_\mathrm{e}^*}{\mathrm{q}_\mathrm{e}} \mu$
- τ typically about 0.9 ps.
- $\delta \mathrm{t}$ drawn from exponential distribution with mean of τ
- $\mathrm{V}_\mathrm{th} = \sqrt{\frac{3\mathrm{kT}}{2\mathrm{m}_\mathrm{e}^*}}$
- $\mathrm{V}_\mathrm{th} \approx \mu \mathrm{E}$
- Each thermal step in a random direction in 3 dimensions.
- Typically about 1000 steps to propagate to the collecting well.

Diffusion Model Check-out - Step Function



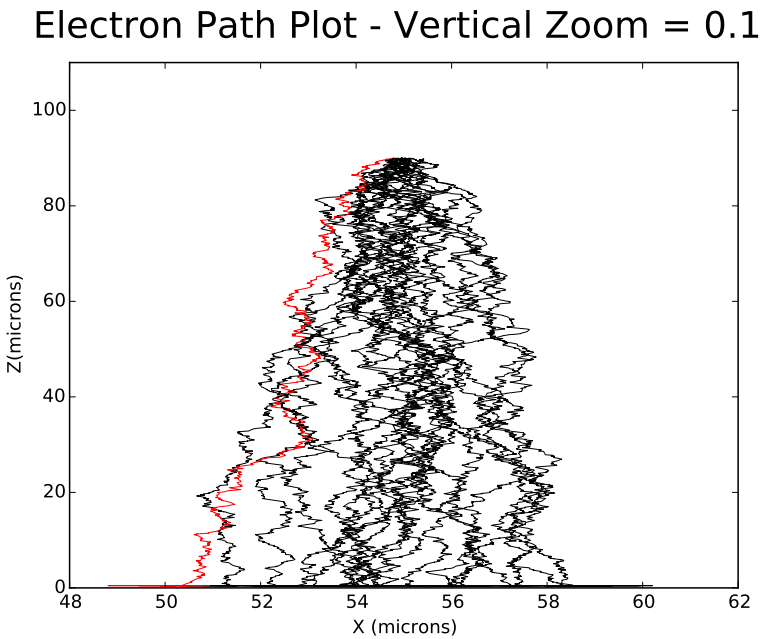


10^8 electrons in initial step function - no E-field.

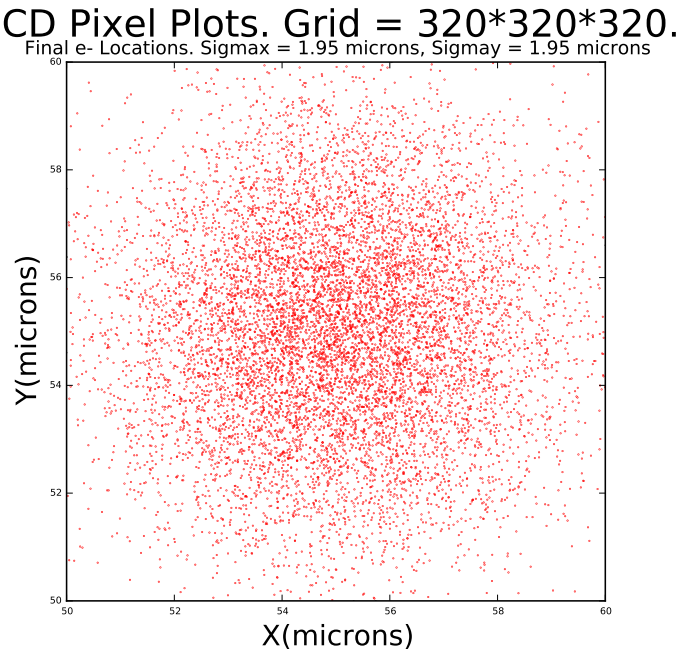
$$\mathrm{erf}\left(\frac{\mathrm{x}-\mathrm{x}_1}{\sqrt{4\mathrm{D}\tau\mathrm{N}_{\mathrm{steps}}}}\right) - \mathrm{erf}\left(\frac{\mathrm{x}-\mathrm{x}_2}{\sqrt{4\mathrm{D}\tau\mathrm{N}_{\mathrm{steps}}}}\right)$$

$$\mathrm{D} = \frac{\mathrm{kT}}{\mathrm{q}}\mu$$

Diffusion Model Check-out - Small (0.1 micron) Spot



Electron Paths

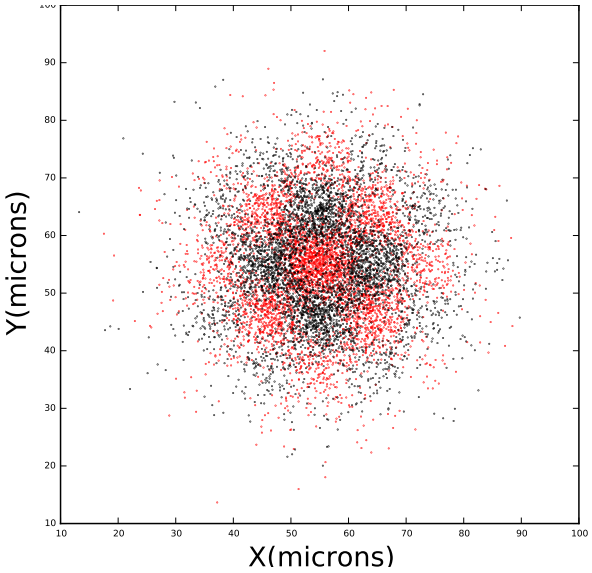


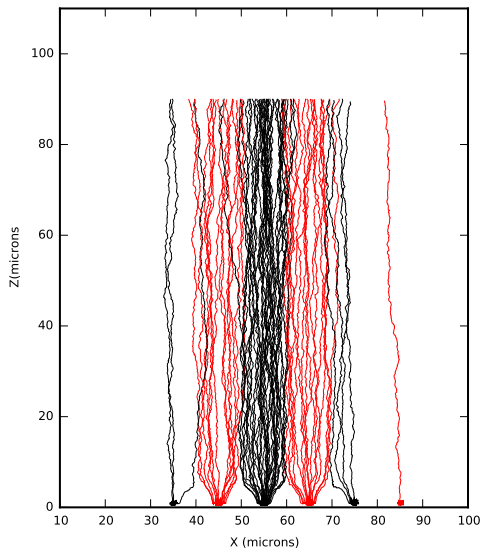
Spot size at collection.

$$\sigma = \sqrt{2 * \mathrm{D} * \mathrm{T}_{\mathrm{transit}}} = \sqrt{2 * \frac{\mathrm{kT}}{\mathrm{q}}\mu * \frac{\mathrm{T}_{\mathrm{Si}}^2}{\mu \mathrm{V}}} = \mathrm{T}_{\mathrm{Si}}\sqrt{\frac{2\frac{\mathrm{kT}}{\mathrm{q}}}{\mathrm{V}}} = 1.95\mu$$

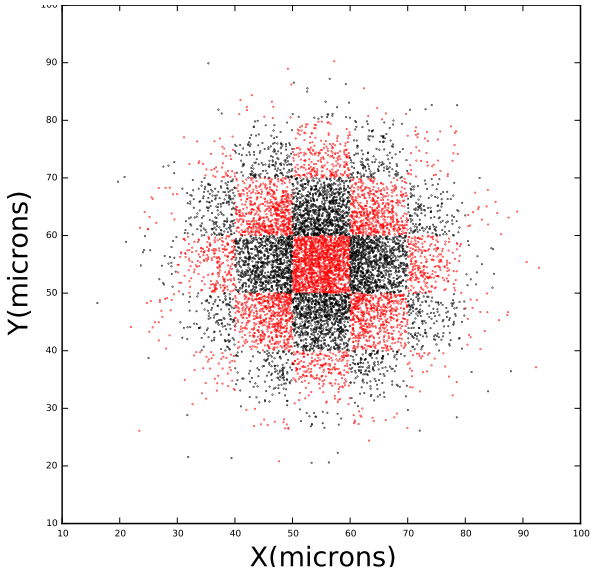
FWHM $= 4.6$ microns, in agreement with Fe measurements.

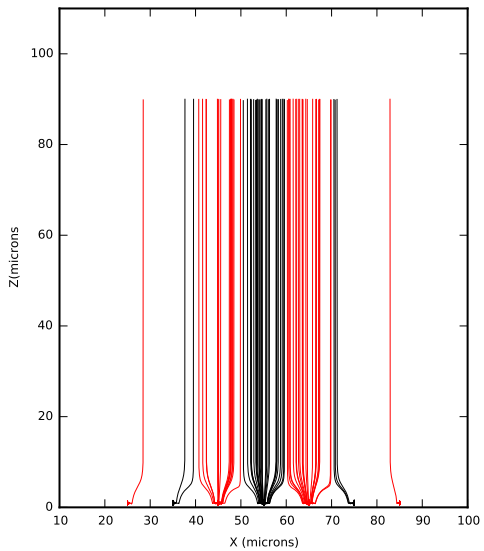
Impact of electron diffusion





Theoretical Diffusion



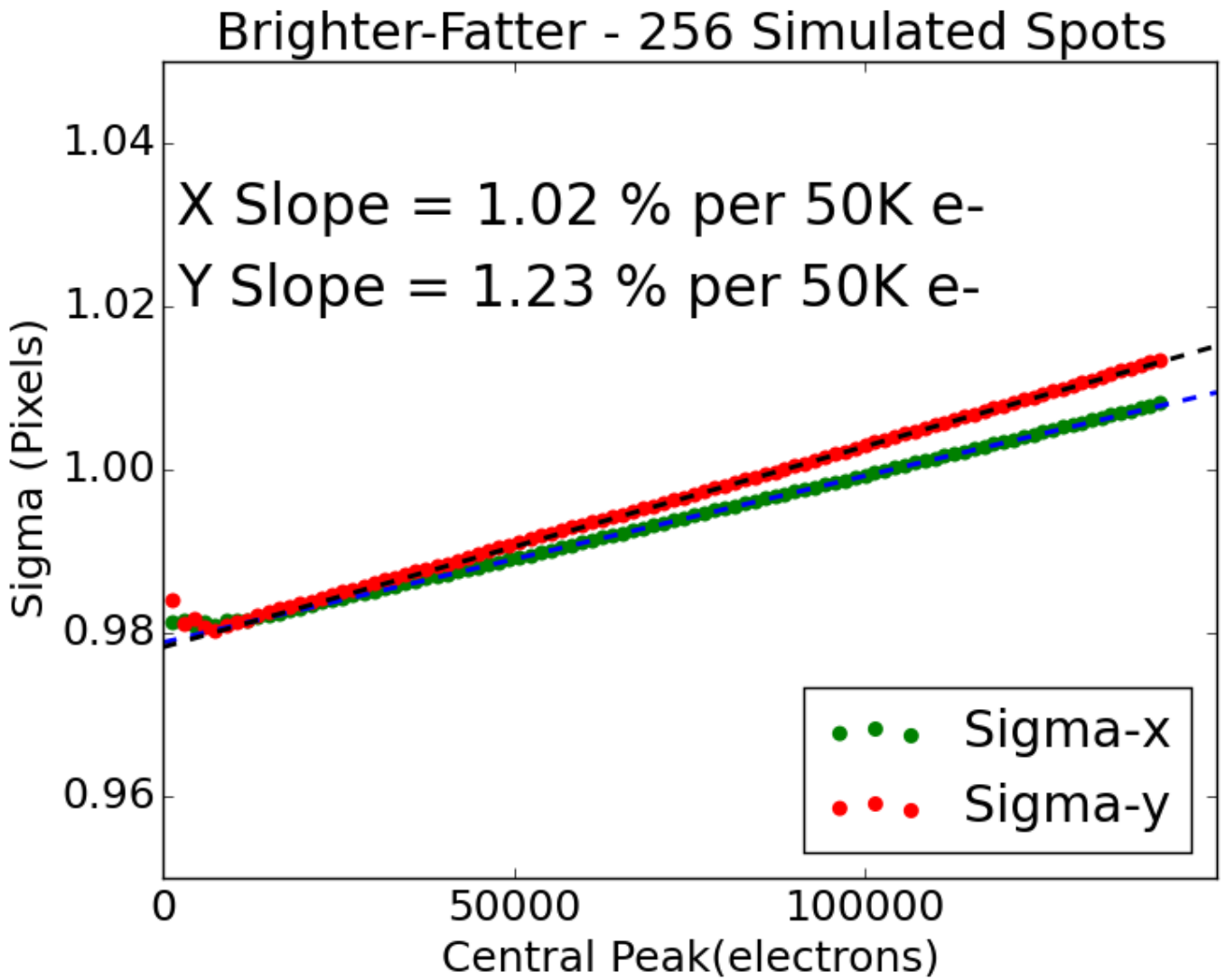


Diffusion turned off

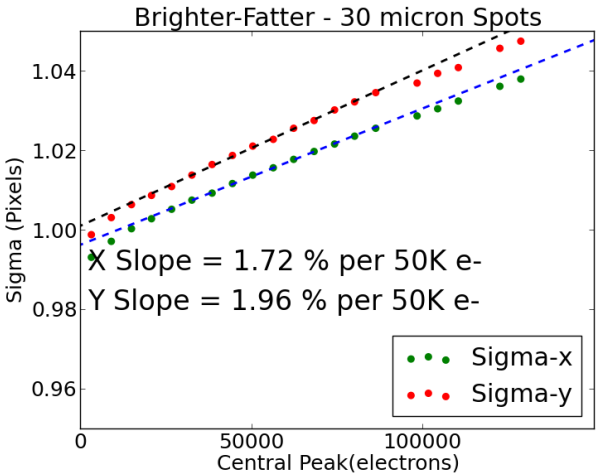
Simulation Strategy for B-F effect.

- Solve Poisson's equation for postage stamp with all pixels empty.
- Choose a random location within the central pixel.
- Determine starting locations for N electrons in a 2D Gaussian spot.
- Propagate these electrons down to their collecting gates.
- Re-solve Poisson's equation with these wells now containing the appropriate charge.
- Repeat with N more electrons.
- I have been using 10,000 electrons per step, which places about 1000 electrons in the central pixel, so about 100 iterations are needed to fill the central pixel.
- In practice, repeat for more than one spot (typical 256), each with a different central location.

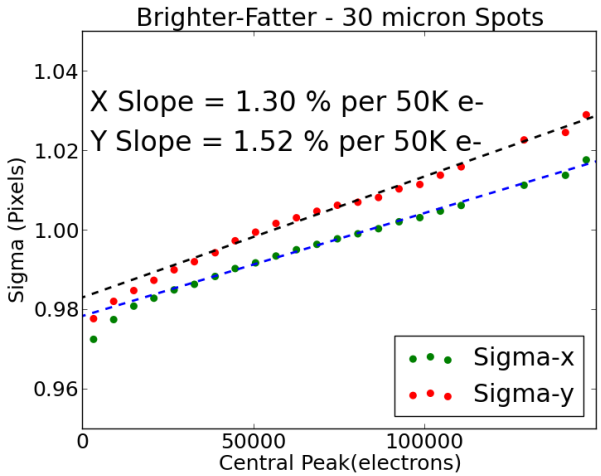
Typical Simulation - 256 Spots - Forward modeled with same code as measurements.



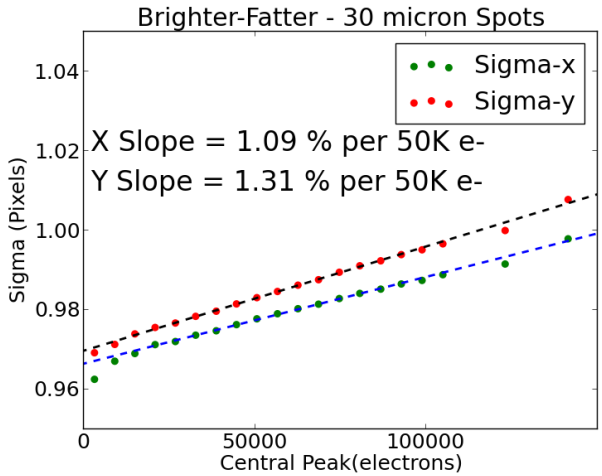
B-F Slopes vs VBB, Measurements and Simulations - Assumed Charge Location



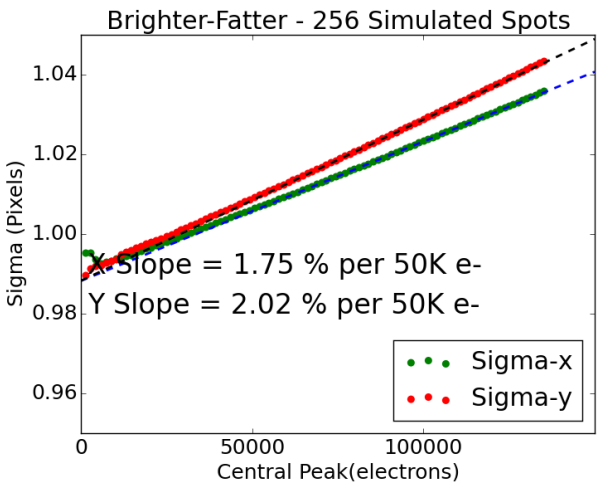
VBB:-30V - Measured



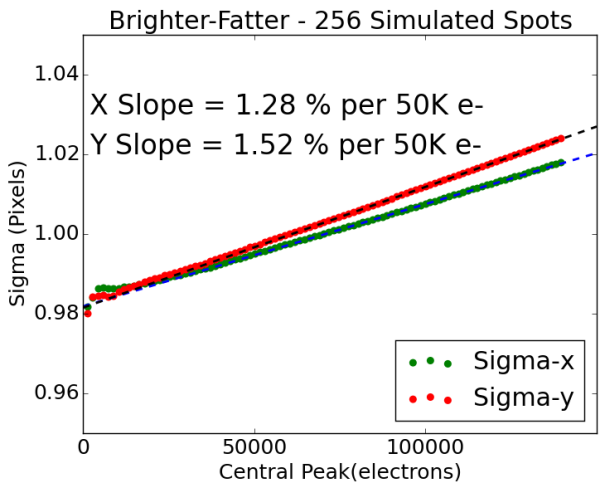
VBB:-45V - Measured



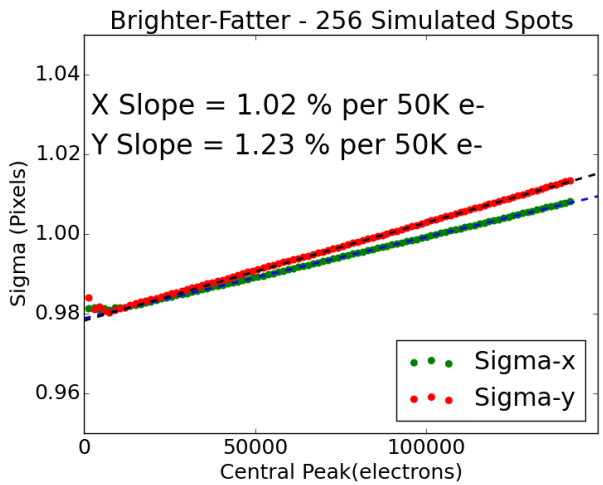
VBB:-60V - Measured



VBB:-30V - Simulated

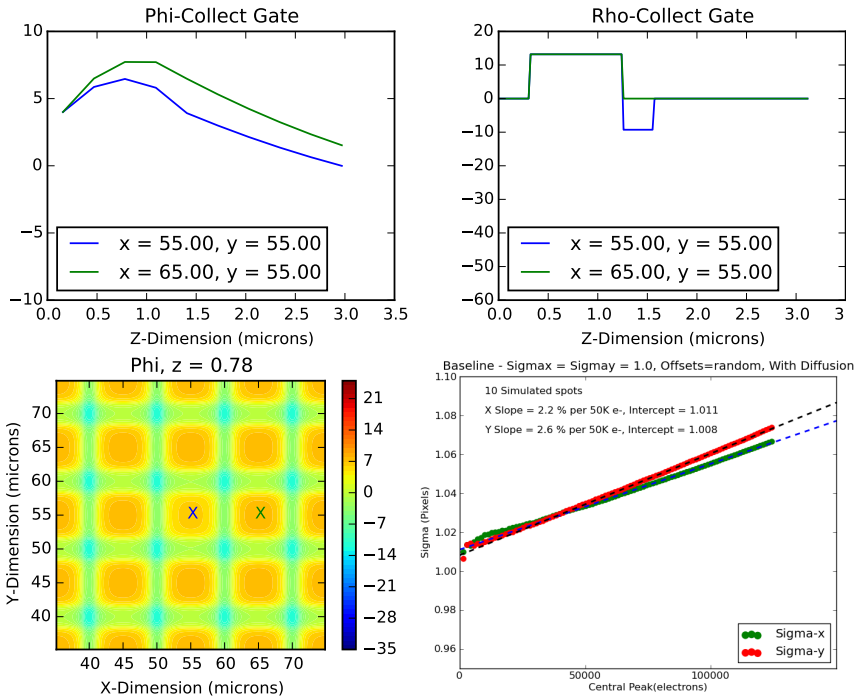


VBB:-45V - Simulated



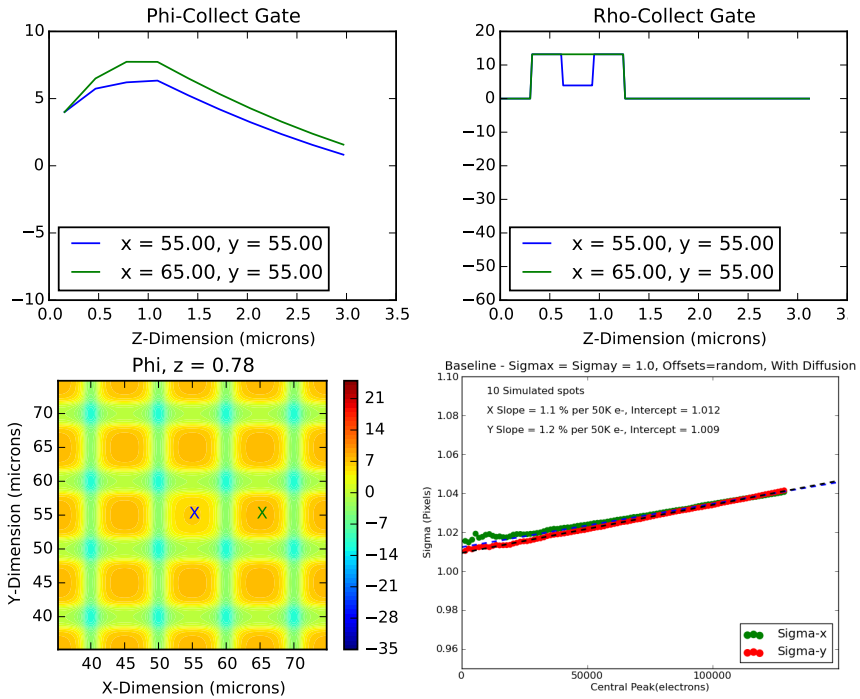
VBB:-60V - Simulated

Vertical location of collected charge impacts BF slopes



Collected Charge at z = 1.4 um

Slopes = 2.2, 2.6% / 50k e-



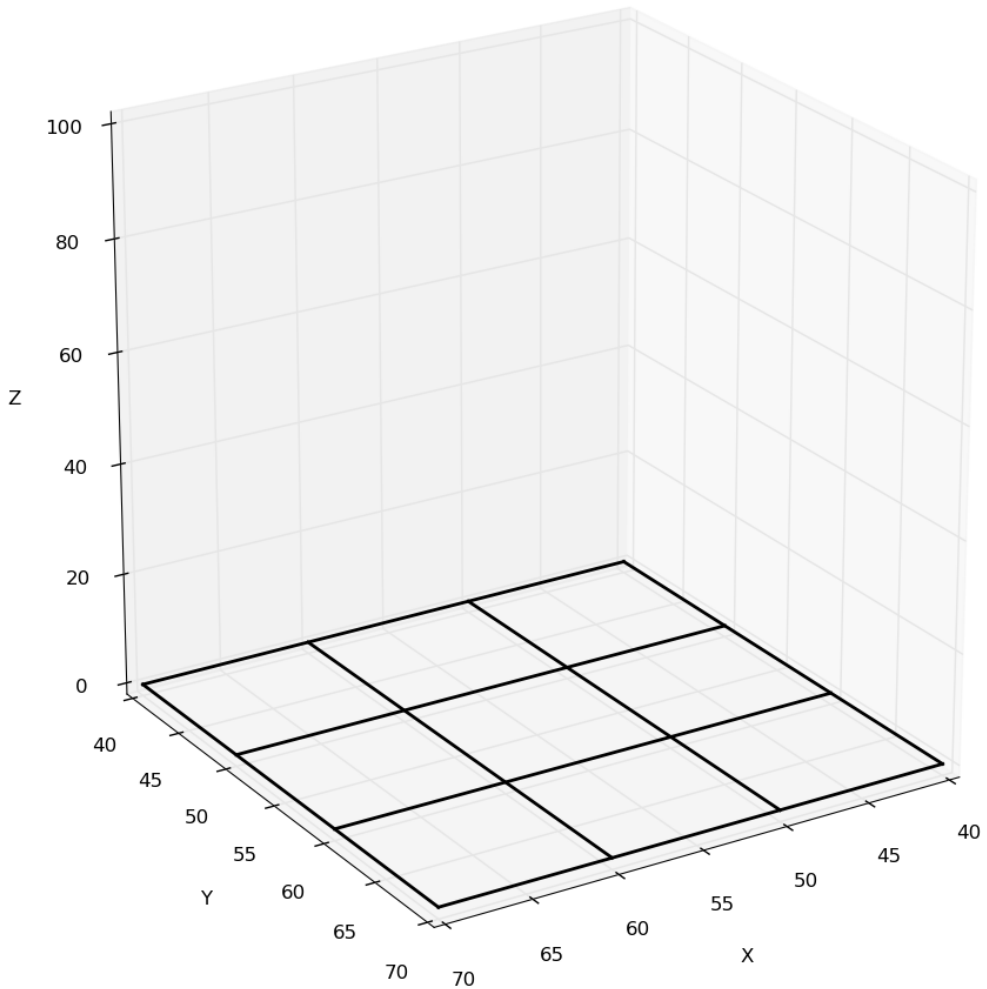
Collected Charge at z = 0.8 um

Slopes = 1.1, 1.2% / 50k e-

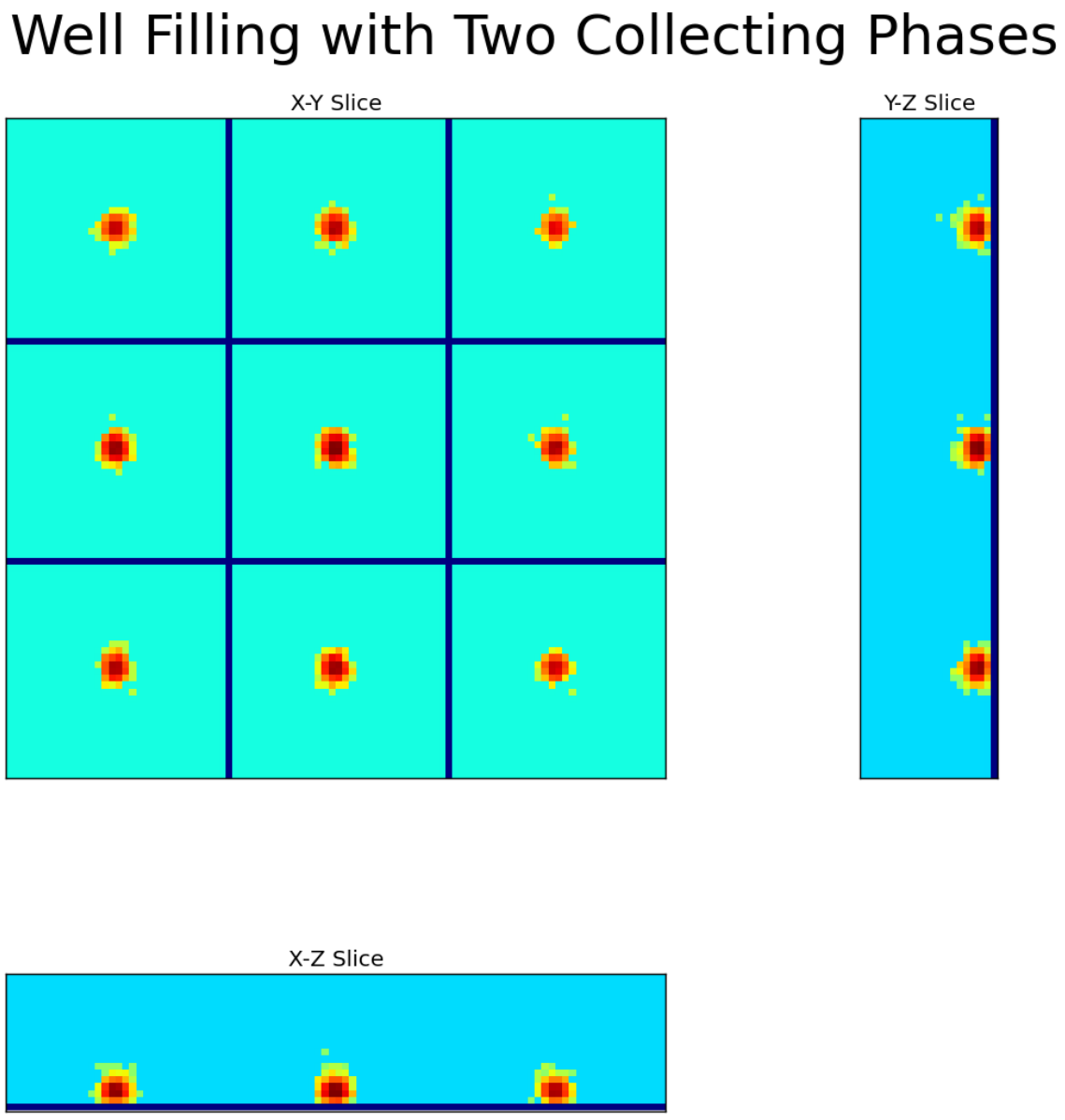
Self-Consistent Stored Charge Locations

- After propagating electron down to collecting well, we know it's location in three dimensions.
- Continue stepping for some time (100-1000 scattering times) after reaching the collecting well to make sure location has converged.
- Keep this location and use it to calculate the new potentials.

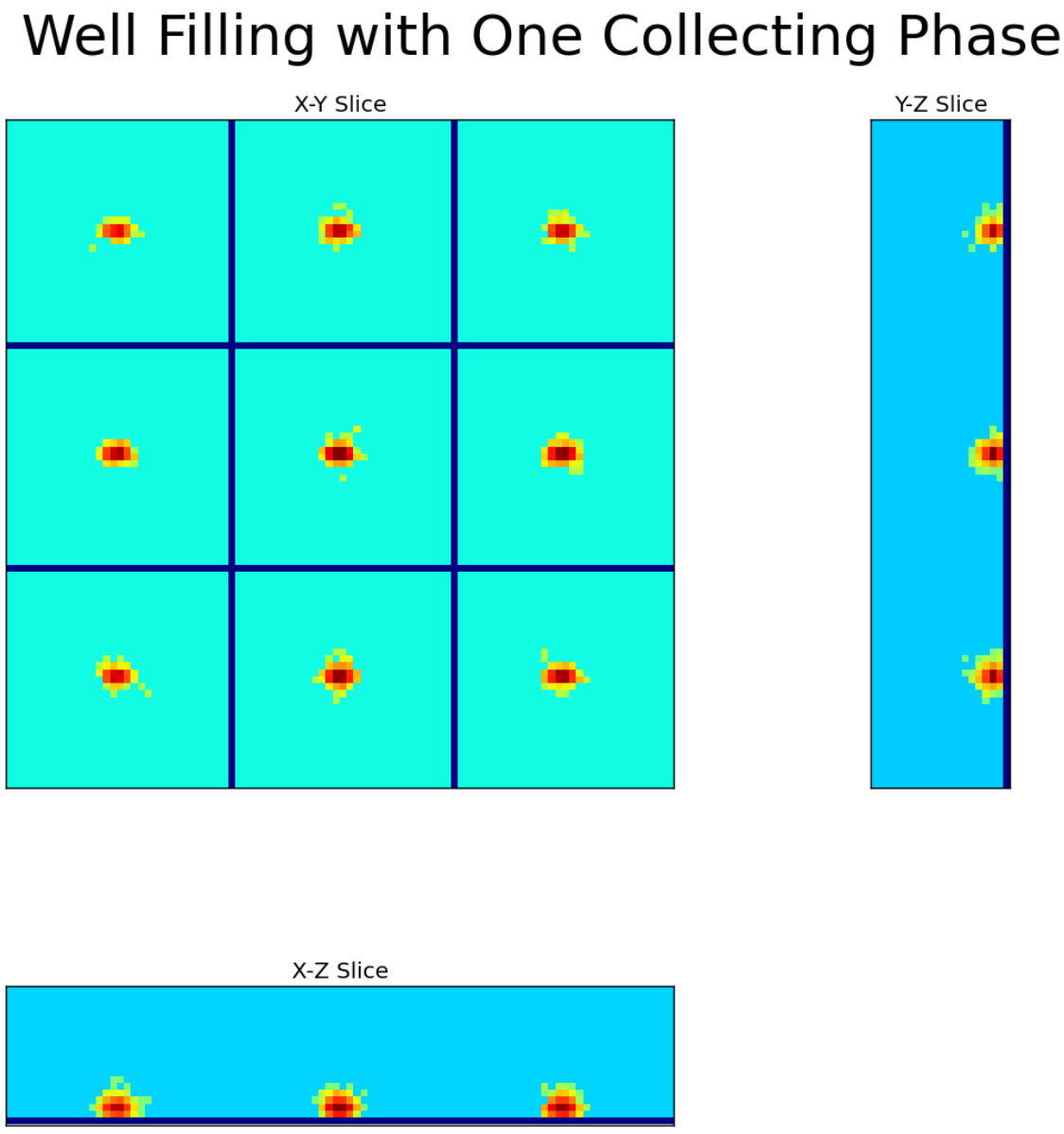
Movie of Pixel Filling - First 10,000 Electrons



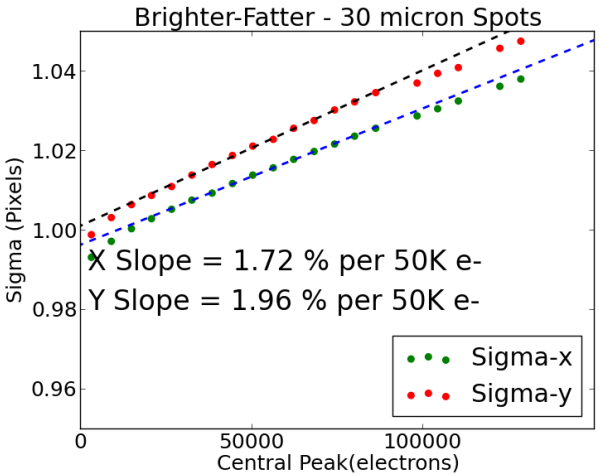
Movie of Pixel Filling - Two Collecting Phases



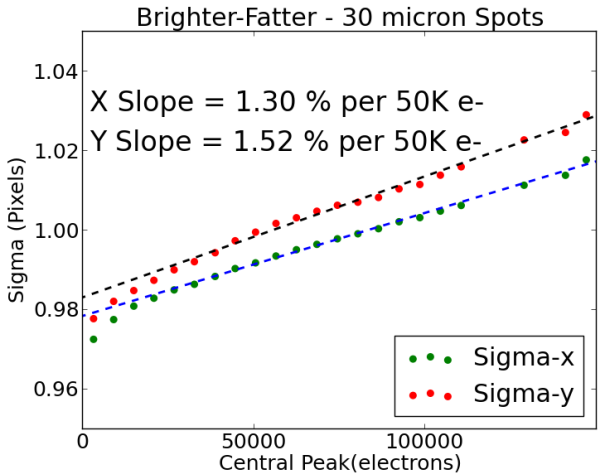
Movie of Pixel Filling - One Collecting Phase



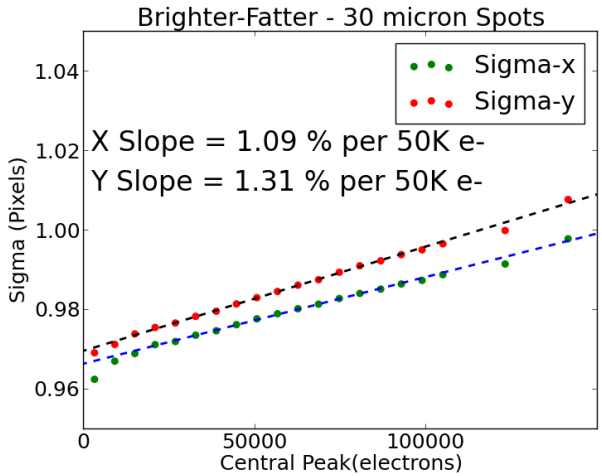
B-F Slopes vs VBB, Measurements and Simulations - Self-Consistent Charge Location



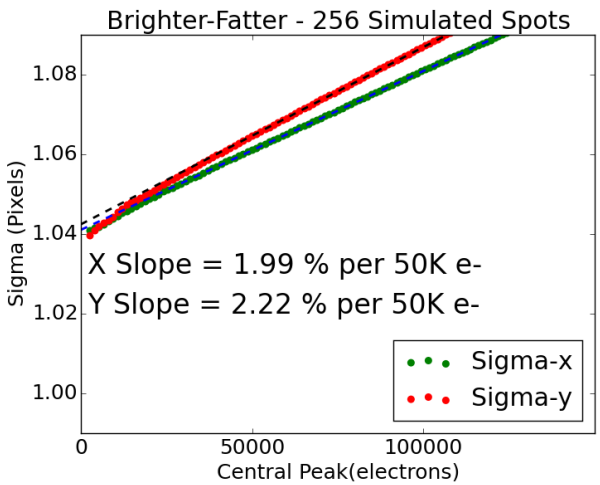
VBB:-30V - Measured



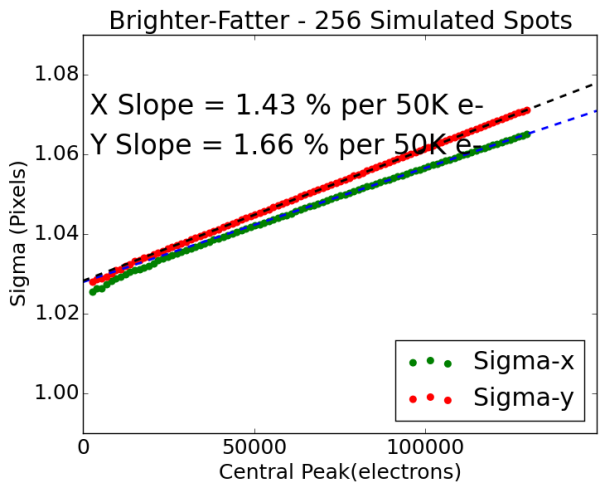
VBB:-45V - Measured



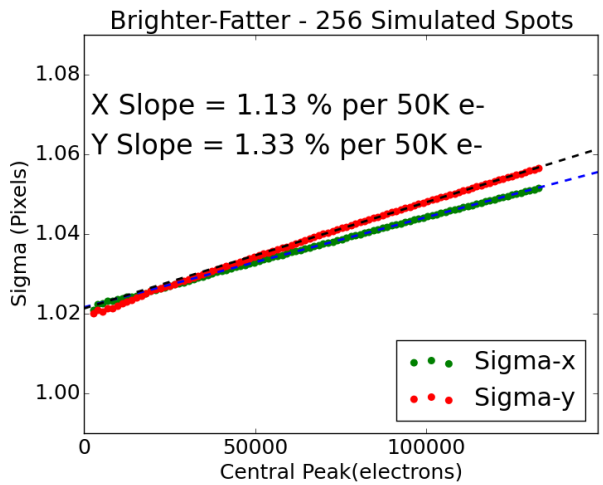
VBB:-60V - Measured



VBB:-30V - Simulated



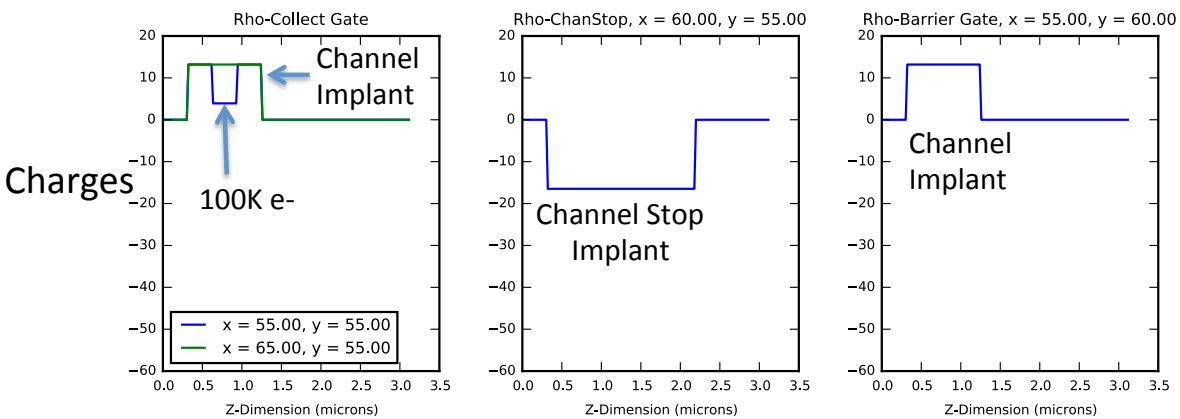
VBB:-45V - Simulated



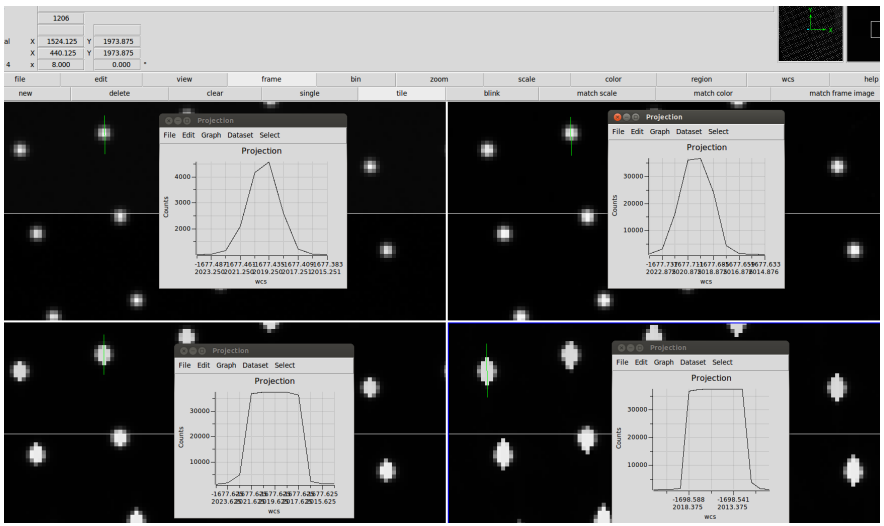
VBB:-60V - Simulated

What are the Free Parameters?

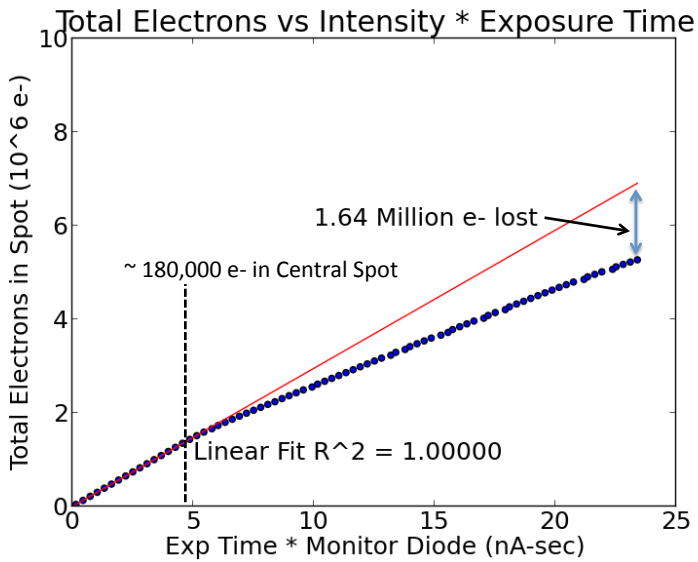
- Diffusion Model:
  - None - Well established Silicon parameters.
- Potentials at Boundaries:
  - None - Applied voltages and geometries are known.
- Charges in Silicon Bulk:
  - Total Charge, Depth, and Profile in Channel region
  - Total Charge, Depth, and Profile in Channel Stop region
  - Is Channel Stop region depleted or are there free holes?
  - Can a device simulator like Silvaco help pin these down?
  - If not, we will tune them in based on CCD measurements.



Measured Saturation Effects

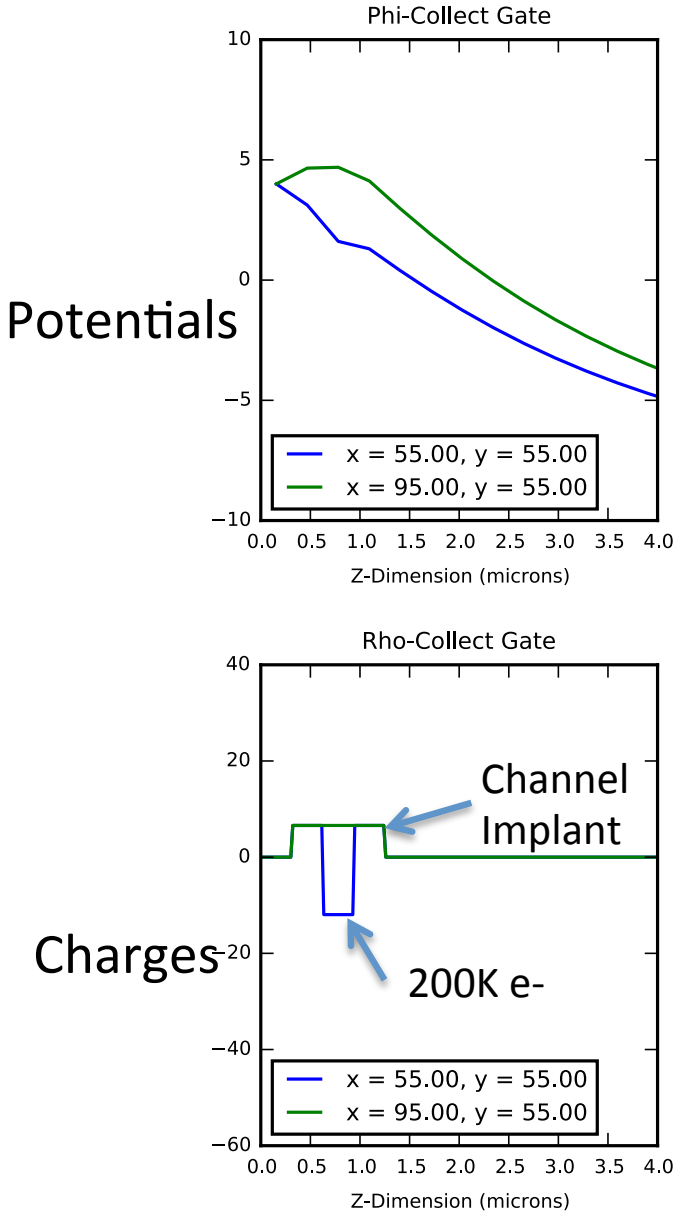


As exposure increases, STA3800 shows saturation at $\approx$ 180,000 electrons/pixel.



Electrons not just being re-distributed, but being lost!

Modeling Saturation Effects



- At $\approx$ 200,000 electrons, buried channel disappears.
- Potential maximum (where electrons go) contacts the surface.
- Subsequent charge is lost, either through trapping or recombination at the surface.

Performance Benchmark

- NERSC Edison - 1 core
- 360^3 grid - grid cell 0.31 μ
- 10,000 electrons

Initialize	Poisson Solution	Calculate E Fields	Trace 10,000 electrons
4 sec.	40 sec.	3 sec.	27 sec.

- So a B-F run with 256 spots, 3 million electrons (300,000 in central spot) takes about 6 hours.

Conclusions and Next Steps

- Conclusions:
  - Simulations are reproducing major aspects of B-F measurements:
    - Magnitude of slopes.
    - Difference between X and Y slopes.
    - Change in slopes with Vbb.
  - While there are some free parameters, we should be able to narrow these down with more knowledge of the CCD and more measurements.
    - Use Silvaco to better determine doping profiles?

- Next Steps
  - More measurements and simulations, especially:
    - Improve modeling of saturation effects.
    - One vs Two collecting phases.
    - Impact of parallel gate voltages.
    - Different spot sizes.
  - Get more people using the code:
    Latest version (https://github.com/craiglagegit/Poisson_CCD16)

Back-Up Slides
Description of Numerics

Solving Poisson's Equation on a Grid

$$\nabla^2 \varphi = \rho$$

$$\frac{\partial^2 \varphi_{i,j,k}}{\partial x^2} = \frac{(\varphi_{i+1,j,k} - \varphi_{i,j,k}) - (\varphi_{i,j,k} - \varphi_{i-1,j,k})}{h^2}$$

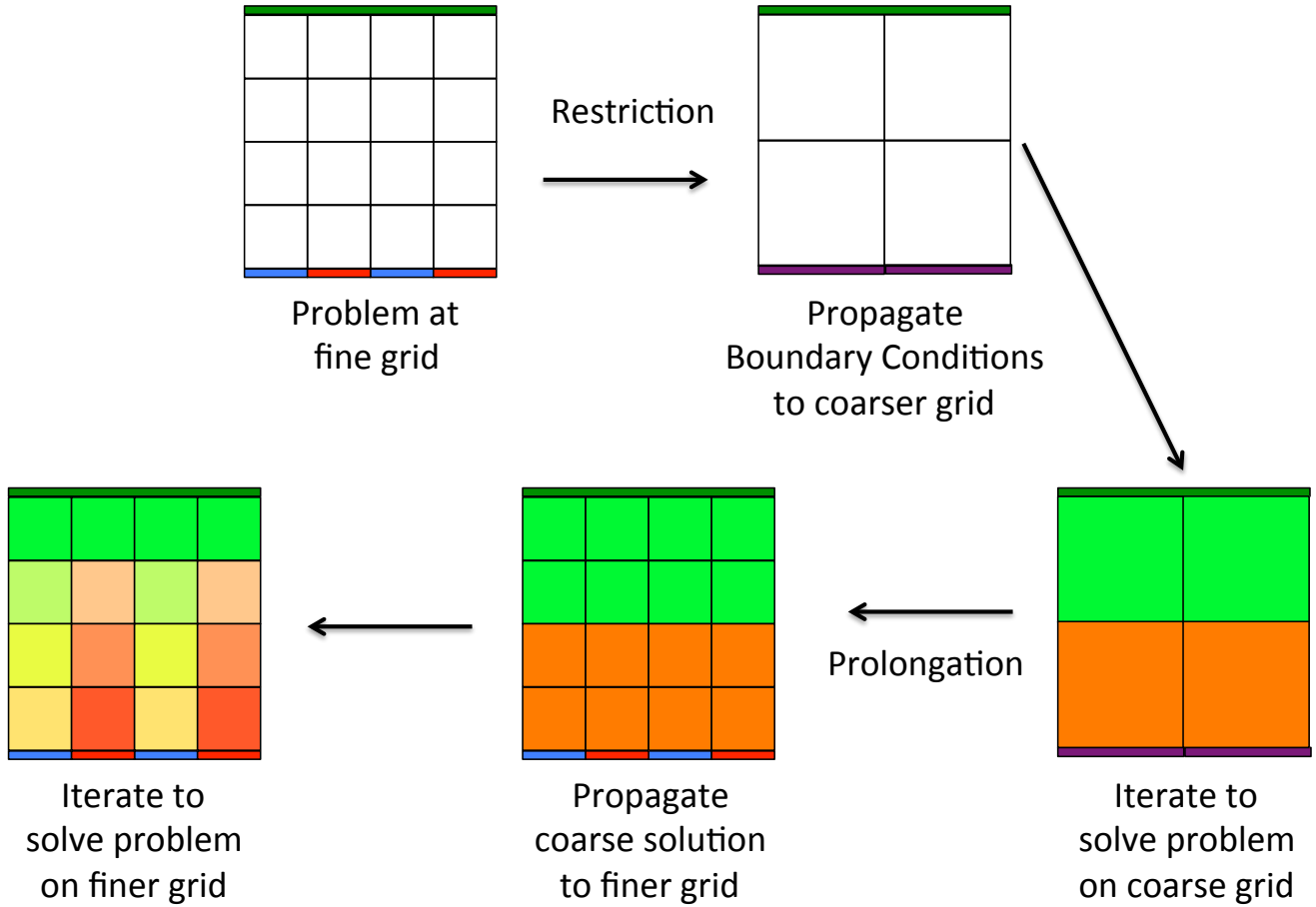
$$(\varphi_{i+1,j,k}+\varphi_{i-1,j,k}+\varphi_{i,j+1,k}+\varphi_{i,j-1,k}+\varphi_{i,j,k+1}+\varphi_{i,j,k-1}-6*\varphi_{i,j,k}) = h^2*\rho_{i,j,k}$$

$$\varphi_{i,j,k} = \frac{1}{6}*(\varphi_{i+1,j,k}+\varphi_{i-1,j,k}+\varphi_{i,j+1,k}+\varphi_{i,j-1,k}+\varphi_{i,j,k+1}+\varphi_{i,j,k-1}-h^2*\rho_{i,j,k})$$

$$\varphi^{(n+1)}_{i,j,k} = \frac{1}{6}*(\varphi^{(n)}_{i+1,j,k}+\varphi^{(n)}_{i-1,j,k}+\varphi^{(n)}_{i,j+1,k}+\varphi^{(n)}_{i,j-1,k}+\varphi^{(n)}_{i,j,k+1}+\varphi^{(n)}_{i,j,k-1}-h^2*\rho_{i,j,k})$$

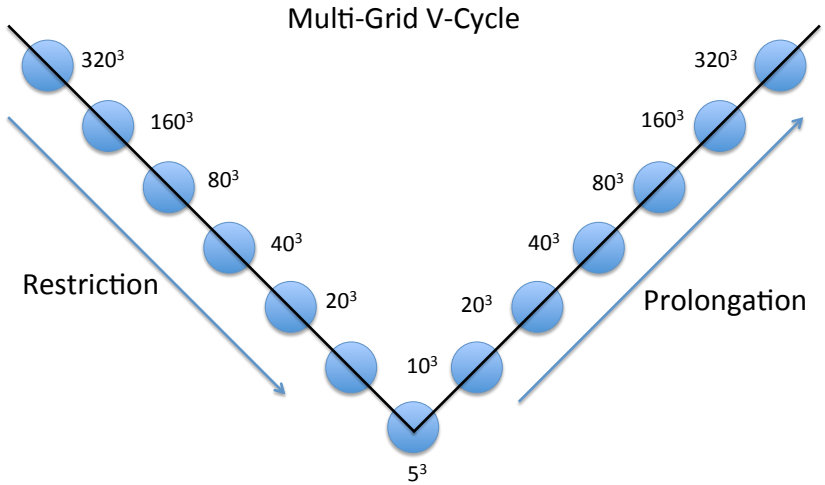
- Conceptually, we simply iterate until convergence.
- In practice, it converges very slowly - millions of iterations are required.

Multi-Grid Methods to the Rescue - I



- Long wavelength modes are determined at the coarse grid.
- Iterations at each finer grid only need to be long enough to determine the short wavelength modes.

Multi-Grid Methods to the Rescue - II



Finest Grid	Cells/Pixel	Grid Spacing	Time (laptop)
160^3	16	0.625 micron	5 sec.
320^3	32	0.3125 micron	40 sec.
640^3	64	0.15625 micron	5 min.

- Each successive step down is ≈ 8 times faster than the next larger grid.
- In practice, I iterate the coarsest grid to machine precision, then 2X fewer iterations at each finer grid, ending with 128 iterations at the finest grid.